Long Calculus II

Name:___________________

Instructor:______________

Final Exam

Spring 2011

Circle your answer below

(1) [a] [b] [c] [d] [e]    (16) [a] [b] [c] [d] [e]
(2) [a] [b] [c] [d] [e]    (17) [a] [b] [c] [d] [e]
(3) [a] [b] [c] [d] [e]    (18) [a] [b] [c] [d] [e]
(4) [a] [b] [c] [d] [e]    (19) [a] [b] [c] [d] [e]
(5) [a] [b] [c] [d] [e]    (20) [a] [b] [c] [d] [e]
(6) [a] [b] [c] [d] [e]    (21) [a] [b] [c] [d] [e]
(7) [a] [b] [c] [d] [e]    (22) [a] [b] [c] [d] [e]
(8) [a] [b] [c] [d] [e]    (23) [a] [b] [c] [d] [e]
(9) [a] [b] [c] [d] [e]    (24) [a] [b] [c] [d] [e]
(10) [a] [b] [c] [d] [e]   (25) [a] [b] [c] [d] [e]
(11) [a] [b] [c] [d] [e]
(12) [a] [b] [c] [d] [e]
(13) [a] [b] [c] [d] [e]
(14) [a] [b] [c] [d] [e]
(15) [a] [b] [c] [d] [e]
1) A farmer has 200yd of fence to be used in constructing three sides of a rectangular pen; an existing long straight wall is to be used for the fourth side of the pen. What is the size of the maximal area of the pen?

\[ \text{[a] 4500} \quad \text{[b] 5000} \quad \text{[c] 5500} \quad \text{[d] 6000} \quad \text{[e] None of the above} \]

2) The absolute maximum value of the function \( f(x) = 2x^3 - 3x^2 - 12x + 15 \) on the closed interval \([0,3]\) is

\[ \text{[a] 22} \quad \text{[b] 11} \quad \text{[c] 6} \quad \text{[d] 15} \quad \text{[e] None of the above} \]

3) If \( \int_{-1}^{1} f(x) \, dx = 3 \), \( \int_{2}^{3} f(x) \, dx = -2 \) and \( \int_{1}^{3} f(x) \, dx = 5 \), then \( \int_{1}^{2} f(x) \, dx = \)

\[ \text{[a] 0} \quad \text{[b] 5} \quad \text{[c] 8} \quad \text{[d] 10} \quad \text{[e] None of the above} \]

4) If \( \int_{1}^{2} (\ln x) / x \, dx = 0.5 \), then \( c = \)

\[ \text{[a] 2} \quad \text{[b] e} \quad \text{[c] 4} \quad \text{[d] e/2} \quad \text{[e] None of the above} \]

5) At how many numbers \( c \) does the function \( f(x) = x^3 - x^2 - x + 1 \), \(-1 \leq x \leq 2\), have a tangent line which is parallel to the secant line through the endpoints of the graph of \( f \)?

\[ \text{[a] 0} \quad \text{[b] 1} \quad \text{[c] 2} \quad \text{[d] 3} \quad \text{[e] None of the above} \]

6) The absolute minimum of the function \( f(x) = x^{\frac{2}{3}}(5 - 2x) \) on the interval \([-1,2]\) occurs at \( x = \)

\[ \text{[a] -1} \quad \text{[b] 0} \quad \text{[c] 1} \quad \text{[d] 2} \quad \text{[e] None of the above} \]

7) The total number of inflection points for the function \( f(x) = xe^{\frac{x}{2}} \) is

\[ \text{[a] 0} \quad \text{[b] 1} \quad \text{[c] 2} \quad \text{[d] 4} \quad \text{[e] None of the above} \]
8) The greatest area of a rectangle with sides parallel to the coordinates axes that can be inscribed in the ellipse \(x^2 + 4y^2 = 4\) is

\[ [a] 2\sqrt{2} \quad [b] 4 \quad [c] \frac{3\pi}{2} \quad [d] \frac{\sqrt{17}}{4} \quad [e] \text{ None of the above} \]

9) For how many numbers \(a\) does \(\lim_{x \to 0} \frac{\sin x}{x^3} - \frac{a}{x^2}\) exist?

\[ [a] 1 \quad [b] 2 \quad [c] 3 \quad [d] 4 \quad [e] \text{ None of the above} \]

10) The function \(f(x) = x^2e^x\) is decreasing on the interval

\[ [a] \left[ -\frac{3}{2}, -\frac{1}{8} \right] \quad [b] \left[ -\frac{4}{5}, \frac{3}{2} \right] \quad [c] [-2, 0] \quad [d] \left[ -\frac{9}{4}, -\frac{1}{8} \right] \quad [e] \text{ None of the above} \]

11) The total number of local extreme points of the function \(f(x) = 3x^4 - 4x^3\) is equal to

\[ [a] 0 \quad [b] 1 \quad [c] 2 \quad [d] 3 \quad [e] \text{ None of the above} \]

12) \(\lim_{x \to \infty} (1 - \frac{1}{x})^x =

\[ [a] 1 \quad [b] -1 \quad [c] e \quad [d] \frac{1}{e} \quad [e] \text{ None of the above} \]

13) \(\int_1^{-2} |x| \, dx =

\[ [a] -\frac{5}{2} \quad [b] 3 \quad [c] -3 \quad [d] -\frac{7}{4} \quad [e] \text{ None of the above} \]

14) If \(F(x) = \int_0^{x^2} t \sin t^3 \, dt\), then \(F'(x) =

\[ [a] x^2 \sin x^6 \quad [b] x^3 \sin x^3 \quad [c] x^3 \sin x^5 \quad [d] 2x^3 \sin x^6 \quad [e] \text{ None of the above} \]

15) \(\sum_{k=1}^{4} (k^2 - 3k) =

\[ [a] -5 \quad [b] 0 \quad [c] 15 \quad [d] 30 \quad [e] \text{ None of the above} \]
16) The area of the region between the x-axis and the graph of \( f(x) = x^3 - x^2 - 2x, \) \(-1 \leq x \leq 2\) is

[a] \( \frac{17}{6} \)  [b] \( \frac{52}{17} \)  [c] \( \frac{11}{4} \)  [d] \( \frac{37}{12} \)  [e] None of the above

17) \( \int_{-1}^{1} 3x^2 \sqrt{x^3 + 1} \, dx = \)

[a] \( \frac{4\sqrt{2}}{3} \)  [b] \( \frac{\sqrt{2}}{4} \)  [c] \( \frac{\sqrt{3}}{2} + 1 \)  [d] \( \sqrt{6} - 4 \)  [e] None of the above

18) The total number of critical points for the function \( f(x) = 4x^{\frac{5}{3}} - 8x^{\frac{1}{3}}, x > 0 \) is equal to

[a] 0  [b] 1  [c] 2  [d] 3  [e] None of the above

19) For which number \( a \) does the limit \( \lim_{x \to 0} \frac{e^x - 1}{\sin ax} \) not exist?

[a] 1  [b] \( \pi \)  [c] \( \frac{\pi}{2} \)  [d] \( 2\pi \)  [e] None of the above

20) \( \int_{0}^{\pi} \frac{\cos 2x}{1 + \sin 2x} \, dx = \)

[a] \( \sqrt{2} \)  [b] \( \frac{1}{2} \ln 2 \)  [c] \( \frac{1}{2} \sqrt{2} \)  [d] \( \ln \pi \)  [e] None of the above

21) A particle starts from rest (that is, with initial velocity zero) at the point \( x = 10 \) and moves along the x-axis with the acceleration function \( a(t) = 12t. \) The resulting position function is \( x(t) = \)

[a] \( 6t^2 + 10 \)  [b] \( 3t^2 + t + 20 \)  [c] \( 2t^3 + 10 \)  [d] \( 10t + 12 \)  [e] None of the above

22) The average value of the function \( f(x) = x \sin x^2 \) on the interval \( [0, \sqrt{\pi}] \) is equal to

[a] \( \sqrt{\pi} \)  [b] \( \frac{1}{\sqrt{\pi}} \)  [c] \( \pi^2 \)  [d] \( \frac{1}{\pi} \)  [e] None of the above

23) The area bounded by the line \( y = 3x \) and the curve \( y = x^3 + 2x^2 \) is equal to
\[ \text{[a] } \frac{71}{6} \quad \text{[b] } \frac{46}{7} \quad \text{[c] } \frac{9}{2} \quad \text{[d] } \frac{51}{5} \quad \text{[e] None of the above} \]

24) \( \lim_{x \to \infty} \frac{\ln x}{\ln(1+e^x)} = \)

\[ \text{[a] } e \quad \text{[b] } \frac{1}{e} \quad \text{[c] } \frac{1}{1+e} \quad \text{[d] } 0 \quad \text{[e] None of the above} \]

25) How many critical points does the function \( f(x) = \frac{(x+1)^2}{1+x^2} \) have?

\[ \text{[a] } 0 \quad \text{[b] } 1 \quad \text{[c] } 2 \quad \text{[d] } 3 \quad \text{[e] None of the above} \]