

MATH 121 FINAL EXAM

Name: _____ Section Number: _____ Instructor: _____

Tuesday, December 11, 2007 8:00 AM — 12:00 NOON

1. Do NOT SEPARATE answer page from rest of test.
2. Work and CIRCLE the answer to each problem INSIDE this test.
3. Circle your answers a SECOND time on this page.
4. One choice for one problem; no penalty for wrong answers.
5. Each problem carries equal weight: 5 points.
6. The following calculators are allowed: TI-82, 83 plus, 84 plus, 85 and 86; Calculators with symbolic manipulation capabilities are NOT allowed.

QUESTION	ANSWER	QUESTION	ANSWER
1.	a b c d e	19.	a b c d e
2.	a b c d e	20.	a b c d e
3.	a b c d e f	21.	a b c d e
4.	a b c d e f	22.	a b c d e
5.	a b c d e	23.	a b c d e
6.	a b c d e	24.	a b c d e
7.	a b c d	25.	a b c d e
8.	a b c d e	26.	a b c d e
9.	a b c d e	27.	a b c d e
10.	a b c d e	28.	a b c d
11.	a b c d e	29.	a b c d
12.	True False	30.	a b c d e f
13.	a b c d e	31.	a b c d
14.	a b c d e		
15.	a b c d e		
16.	a b c d e		
17.	a b c d e		
18.	a b c d		

1. What is the domain of the function

$$f(x) = \frac{1}{(1+x)(1-e^x)}$$

- (a) All reals.
- (b) All reals except the number -1 .
- (c) All reals except 0 .
- (d) All reals except 0 and -1 .
- (e) All reals except 1 and -1 .

2. Simplify the expression $\tan(\cos^{-1}(x))$

- (a) $\frac{\tan x}{\cos x}$.
- (b) $\frac{\sqrt{1-x^2}}{x}$.
- (c) $\frac{x}{\sqrt{1-x^2}}$
- (d) x .
- (e) $\tan\left(\frac{1}{\cos x}\right)$

3. If it exists as a finite number or infinity, find

$$\lim_{x \rightarrow 1^-} \frac{1}{x-1} - \frac{1}{|x-1|}$$

- (a) Does not exist.
- (b) $\infty - \infty$.
- (c) ∞
- (d) $-\infty$.
- (e) 0 .
- (f) 1 .

4. If it exists as a finite number or infinity, find

$$\lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right).$$

- (a) Does not exist.
- (b) $\infty - \infty$.
- (c) ∞ .
- (d) $-\infty$.
- (e) 0.
- (f) 1

5. Find

$$\lim_{x \rightarrow -2^+} \frac{x^2 - 4}{|x + 2|}.$$

- (a) 4.
- (b) -4.
- (c) 2.
- (d) 0
- (e) ∞

6. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

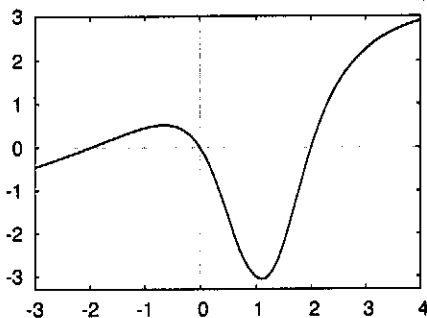
$$f(x) = \begin{cases} cx^3 + 3x & \text{if } x < -1 \\ cx^2 & \text{if } x \geq -1 \end{cases}$$

- (a) There is no such c .
- (b) 3.
- (c) $-3/2$.
- (d) Any c is good.
- (e) $3x/(x^2 - x^3)$.

7. Which of the following statements is true?

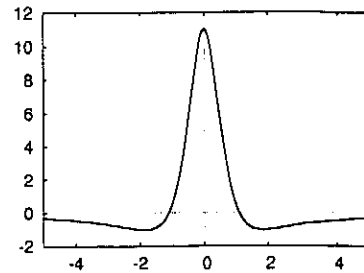
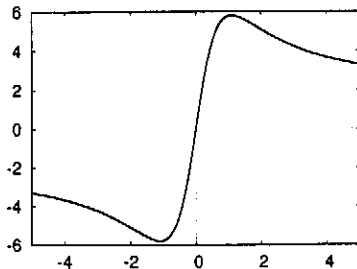
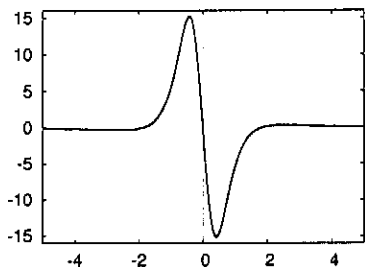
- (a) The graph of $y = f(x)$ **can** intersect its asymptotes — both vertical and horizontal ones
- (b) The graph of $y = f(x)$ **can** intersect its vertical asymptotes but **cannot** intersect its horizontal asymptotes
- (c) The graph of $y = f(x)$ **can** intersect its horizontal asymptotes but **cannot** intersect its vertical asymptotes
- (d) The graph of $y = f(x)$ **cannot** intersect its asymptotes — neither vertical nor horizontal ones

8. For the function g whose graph is given, which inequality is true?



- (a) $g'(0) < 0 < g'(-1) < g'(3) < g'(2)$
- (b) $g'(-1) < 0 < g'(0) < g'(2) < g'(3)$
- (c) $g'(0) < g'(-1) < 0 < g'(3) < g'(2)$
- (d) $g'(0) < 0 < g'(-1) < g'(2) < g'(3)$
- (e) none of the above

9. The figures show the graphs of f , its first and second derivatives in the following order (from left to right)



- (a) f, f', f''
 (b) f', f, f''
 (c) f'', f, f'
 (d) f, f'', f'
 (e) none of the above
10. The graph of $f(x) = x^3 + 3x^2 + x + 3$ has a horizontal tangent at

- (a) $x = -3$
 (b) $x = -1 + \sqrt{\frac{3}{2}}$ and $x = -1 - \sqrt{\frac{3}{2}}$
 (c) $x = -1$
 (d) $x = -3 + \sqrt{\frac{2}{3}}$ and $x = -3 - \sqrt{\frac{2}{3}}$
 (e) none of the above

11. The equation of the tangent line to the graph of $f(x) = \frac{2x}{x+1}$ at the point $(1, 1)$ is

- (a) $y = \frac{2}{(x+1)^2}(x-1) + 1$
 (b) $y = x$
 (c) $y = \frac{x-1}{2}$
 (d) $y = \frac{x+1}{2}$
 (e) none of the above

12. Is the following statement **true** or **false**?

$$\frac{d}{dx}|x^2 + x| = |2x + 1|$$

True False

13. If $f(x) = g(x)^{h(x)}$, and $g(0) = 1, g'(0) = 2, h(0) = 3, h'(0) = 4$, evaluate $f'(0)$.

- (a) 6
- (b) 3
- (c) e^6
- (d) 64
- (e) none of above

14. Given $f(x) = e^{x^2}$, find $f'(x)$?

- (a) $x^2 e^{x^2-1}$
- (b) $2x + e^{x^2}$
- (c) $2xe^{x^2}$
- (d) e^{x^2}
- (e) none of above

15. Find the equation of the tangent line of the curve at $(e, 1)$

$$y = \ln\left(\frac{x}{y}\right)$$

- (a) $y - 1 = \frac{e+1}{e}(x - e)$
- (b) $y - 1 = \frac{1}{e}(x - e)$
- (c) $y - 1 = x - e$
- (d) $y - 1 = \frac{1}{2e}(x - e)$
- (e) none of above

16. The radius r of a sphere changes at a rate of $1/4$ m/s. Find the instantaneous rate of change of the volume of the sphere when $r = 2$.
- (a) $8\pi/3$
 - (b) 16π
 - (c) 8π
 - (d) 4π
 - (e) none of above
17. A bacteria culture grows with constant relative growth rate. The number of bacteria grows from 100 to 2,000 in 6 hours. Find the number of bacteria after 12 hours.
- (a) 40,000
 - (b) 20,000
 - (c) 12,000
 - (d) 4,000
 - (e) none of above
18. A man jogs along a square-shaped field at the speed of 6 miles per hour. Each side of the field is 1 mile long. If he started from a corner of the field, how fast is the distance changing from his current position to that corner after 15 minutes?
- (a) 3.27 miles per hour
 - (b) 4.99 miles per hour
 - (c) -4.99 miles per hour
 - (d) 2.68 miles per hour
19. Given $f(1) = 1$, $f'(1) = 2$, $f(2) = 3$, $f'(2) = 4$ and $g(x) = f(2x)$, use linear approximation to estimate $g(1.1)$.
- (a) 1.4.
 - (b) 3.8.
 - (c) 3.4.
 - (d) 1.2.
 - (e) none of above.

20. The function $f(x) = x^3 - 21x^2 + 80x + 7$ satisfies three conditions of Rolle's Theorem. All the numbers c that satisfy Rolle's Theorem are:

(a) $c_1 = 201 + \frac{\sqrt{7}}{3}$, $c_2 = 201 - \frac{\sqrt{7}}{3}$

(b) $c = 7 - \frac{\sqrt{201}}{3}$

(c) $c_1 = 7 + \frac{\sqrt{201}}{3}$, $c_2 = 7 - \frac{\sqrt{201}}{3}$

(d) $c = 7 + \frac{\sqrt{201}}{3}$

(e) $c = \frac{\sqrt{201}}{3}$

21. Find all the critical points of the function $f(x) = x^4(x - 3)^3$.

(a) $0, 2, \frac{12}{7}$

(b) $0, 3, \frac{12}{11}$

(c) $0, 3, \frac{12}{7}$

(d) $0, 2, \frac{12}{11}$

(e) $0, 2, \frac{7}{12}$

22. Find the point of the line $y = 4x + 8$ that is closest to the origin.

(a) $(\frac{-32}{17}, \frac{10}{17})$

(b) $(\frac{-34}{17}, \frac{9}{17})$

(c) $(\frac{-32}{17}, \frac{8}{17})$

(d) $(-2, \frac{8}{17})$

(e) $(\frac{-31}{17}, \frac{8}{17})$

23. Find the limit.

$$\lim_{t \rightarrow 0} \frac{3^t - 2^t}{t}$$

(a) ∞

(b) 0.

(c) $\ln 2 - \ln 3$

(d) $\ln 3 - \ln 2$

(e) 1

24. How many points of inflection are on the graph of the function $12x^3 + 14x^2 - 7x - 9$?

(a) 3

(b) 1

(c) 4

(d) 2

(e) 5

25. For what values of c does the curve $f(x) = 5x^3 + cx^2 + 10x$ have both a local minimum and a local maximum?

(a) $|c| > 15$

(b) $|c| > \sqrt{150}$

(c) $|c| > 1,500$

(d) $|c| > \sqrt{30}$

(e) $|c| > \sqrt{750}$

26. Determine a definite integral which is equal to the limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{i}{n}\right)^{121} \frac{1}{n}$$

- (a) $\int_1^2 x^{121} dx$
- (b) $\int_0^1 x^{121} dx$
- (c) $\int_{1+\frac{1}{n}}^2 x^{121} dx$
- (d) $\int_{1+\frac{1}{n}}^n x^{121} dx$
- (e) $\int_1^\infty (1+x)^{121} \frac{1}{n} dx$

27. $\frac{d}{dx} \int_0^{1/x} \arctan t dt =$

- (a) $\frac{-1}{x^2(1+x^2)}$
- (b) $\arctan\left(\frac{1}{x}\right)$
- (c) $\frac{-\arctan t}{x^2}$
- (d) $\frac{-\arctan\left(\frac{1}{x}\right)}{x^2}$
- (e) can't do it because we didn't learn how to integrate $\arctan t$.

28. Which of the following statements is NOT true:

- (a) If $1 \leq f(x) \leq 2$ on the interval $[1, 2]$, then $1 \leq \int_1^2 f(x) dx \leq 2$.
- (b) Suppose the length of a rod is l and the linear density function of the rod is $\rho(x)$. Then the total mass of the rod is given by $\int_0^l \rho(x) dx$.
- (c) If $f(x)$ is odd and continuous on the interval $[-1, 1]$, then $\int_{-1}^1 f(x) dx = 0$.
- (d) Suppose a particle is moving on the x-axis and its velocity function is $v(t)$. Then the total distance traveled by the particle in the time interval $[0, 121]$ is equal to $\int_0^{121} v(t) dt$.

29. $\int \left(\frac{5+4x+3x^2}{x^2} + 2 \sin x - 3e^x\right) dx =$

- (a) $\frac{3(5x+2x^2+x^3)}{x^3} - 2 \cos x - 3e^x + C$
- (b) $\frac{-5}{x} + 4 \frac{x^0}{0} + 3x - 2 \cos x - 3e^x + C$
- (c) $\frac{-5}{x} + 4 \ln |x| + 3x + 2 \sin x - 3e^x + C$
- (d) $\frac{-5}{x} + 4 \ln |x| + 3x - 2 \cos x - 3e^x + C$

30. Which of the following is equal to $\int_1^2 3x^2\sqrt{x^3+1} dx$?

(a) $\int_1^2 3(u^2-1)^{2/3}u du$

(b) $\int_2^9 u^{1/2} du$

(c) $\int_1^2 u^{1/2} du$

(d) $\int_1^2 3\sqrt{x^{12}+x^4} dx$

(e) More than one of the above.

(f) None of the above.

31. Let a be a positive constant. Then $\int_0^a 2(x-a)e^{(x-a)^2} dx =$

(a) $1 - e^{a^2}$

(b) $-a^2e^{a^2}$

(c) $2(x-a)(1 - e^{a^2})$

(d) $-a^2(1 - e^{a^2})$