

CALCULUS I 121-01/02

FINAL EXAMINATION

Friday, May 4, 2007, 8:00am - 12 NOON

NAME (Print): _____

RULES:

- You are permitted to have a calculator and writing instruments
- No books or notes allowed
- All the tests are conducted under Tulane honor code; all work to be your own.
- In questions 1–11 choose the correct answer (A, B, C, or D)
- In questions 12–13 put answers in spaces provided
- In questions 14–19 show your work (NO WORK = NO CREDIT), and circle the final answer

1. (3pts) If $f(x) = \frac{3}{x}$ and $g(x) = \frac{2}{x+1}$ find $(f \circ g)(x)$ and its domain.

• $(f \circ g)(x) = \boxed{}$ (A) $\frac{6}{x(x+1)}$ (B) $\frac{2x}{3+x}$ (C) $\frac{6}{x+1}$ (D) $\frac{3}{2}(x+1)$

• $D_{(f \circ g)} : \boxed{}$ (A) \mathbb{R} (B) $x \neq -1$ (C) $x \neq 0$ and $x \neq -1$ (D) $x \neq 0$

2. (2pts) Find the inverse of $f(x) = 4 - 2x^3$.

• $f^{-1}(x) = \boxed{}$ (A) $\sqrt[3]{\frac{4+x}{2}}$ (B) $\left(\frac{4-x}{2}\right)^{-3}$ (C) $\sqrt[3]{\frac{4-x}{2}}$ (D) $4 - 2x^3$

3. (2pts) By definition, the derivative of the function $f(x) = 3x^2 - 4$ is:

• $f'(x) = \boxed{}$

(A) $\lim_{h \rightarrow 0} \frac{(3(x+h)^2 - 4) - (3x^2 - 4)}{h}$

(B) $\lim_{h \rightarrow 0} \frac{(3(x+h)^2 - 4) - (3h^2 - 4)}{h}$

(C) $\lim_{h \rightarrow 0} \frac{(3(x+h)^2 - 4) - (3x^2 - 4)}{x+h}$

(D) $\lim_{h \rightarrow 0} \frac{(3x^2 - 4) - (3h^2 - 4)}{h}$

4. (2pts) Solve for x .

• $\log_5(3x+1) = 4$, $x = \boxed{}$ (A) $\frac{19}{3}$ (B) 208 (C) $\frac{e^4-1}{3}$ (D) 1

• $3^x > 27$ Soln.: $\boxed{}$ (A) $x > 9$ (B) $x > \pm 3$ (C) $x > 3$ (D) $x \geq 3$

5. (2pts) Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3 + x - 1$ on the interval $[-2, 2]$.

• $c = \boxed{}$ (A) $\frac{2}{\sqrt{3}}$ (B) $\approx .681$ (C) 0 (D) $\pm \frac{2}{\sqrt{3}}$

• $f(x) = (\cos x)^x$ $f'(x) = \boxed{}$

(A) $(\cos x)^x \cdot \ln(\cos x)$

(B) $x(\cos x)^{x-1} \cdot (-\sin x)$

(C) $\left(\ln(\cos x) - x \cdot \frac{\sin x}{\cos x}\right) (\cos x)^x$

(D) $\ln(\cos x) - x \cdot \frac{\sin x}{\cos x}$

• $g(x) = \int_0^{\ln x} \sqrt{s^2 + s} ds$ $g'(x) = \boxed{}$

(A) $\sqrt{(\ln x)^2 + \ln x} \cdot \frac{1}{x}$

(B) $\sqrt{(\ln x)^2 + \ln x}$

(C) $\frac{1}{2\sqrt{(\ln x)^2 + \ln x}} \cdot (2\ln x + 1)$

(D) $\frac{1}{2\sqrt{s^2 + s}} \cdot (2s + 1)$

7. (2pts) Find all vertical and horizontal asymptotes of the function $f(x) = \frac{3x+12}{x^2-16}$.

• vertical: $\boxed{}$ (A) $x = -4, +4$ (B) $x = 4$ (C) $x = 0$ (D) *DNE*

• horizontal: $\boxed{}$ (A) $y = -4, +4$ (B) $y = 4$ (C) $y = 0$ (D) *DNE*

8. (8pts) Evaluate the following integrals.

• $\int_1^5 \frac{3}{4x} dx = \boxed{}$ (A) $\frac{18}{25}$ (B) $\frac{3}{4} \cdot \ln 5$ (C) 3 (D) $3 \cdot \ln 5$

• $\int_0^3 (1 + 2y - y^2) dy = \boxed{}$ (A) -3 (B) -6 (C) 0 (D) 3

• $\int_1^e \frac{\ln x}{x} dx = \boxed{}$ (A) $\frac{1}{e}$ (B) $e - 1$ (C) $\frac{1}{2}$ (D) $\frac{e^2}{2} - \frac{1}{2}$

• $\int_0^{\pi/3} \frac{\sin \theta}{\cos^2 \theta} d\theta = \boxed{}$ (A) 1 (B) $\frac{1}{3}$ (C) $\frac{\pi}{3}$ (D) -1

10. (6pts) The position of a particle is given by $s(t) = 2t^3 - 9t^2 + 12t$.

- At what times is the particle at rest ?

$t = \boxed{}$ (A) 0 (B) $\frac{3}{2}$ (C) 1, 2 (D) never

- What is the acceleration of the particle at $t = 1$?

$a = \boxed{}$ (A) 5 (B) -6 (C) 0 (D) -12

- What is the total distance traveled by the particle during the first 3 seconds ?

$d = \boxed{}$ (A) 6 (B) 9 (C) 10 (D) 11

11. (13pts) Find the following limits:

- $\lim_{x \rightarrow 7^-} \frac{9}{x-7} = \boxed{}$ (A) $-\infty$ (B) $+\infty$ (C) 0 (D) 9

- $\lim_{x \rightarrow -2} \frac{2x^2 - 8}{x + 2} = \boxed{}$ (A) 0 (B) 4 (C) -8 (D) -16

- $\lim_{t \rightarrow 0} \frac{t^2}{\sqrt{t^2 + 9} - 3} = \boxed{}$ (A) 0 (B) 6 (C) $+\infty$ (D) $-\infty$

- $\lim_{x \rightarrow +\infty} \frac{7 - 3x + 4x^2}{2x^2 + 10x - 5} = \boxed{}$ (A) $+\infty$ (B) 1 (C) $\frac{7}{2}$ (D) 2

- $\lim_{t \rightarrow -\infty} \frac{3t^6 - 2}{t^5 + 1} = \boxed{}$ (A) $+\infty$ (B) $-\infty$ (C) 3 (D) 0

- $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}} = \boxed{}$ (A) 0 (B) 1 (C) e (D) $+\infty$

14. (4pts) Find the equation of the tangent line to the curve $x^2 + 3xy + y^2 = x^3$ at the point $(1, -3)$.

15. (4pts) Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection ?

16. (4pts) A rectangular storage container with an open top is to have a volume of 36 m^3 . The length of its base is twice the width. Material for the box costs \$10 per square meter. Find the cost of material for the cheapest such container, and also give the dimensions of the container.

17. (5pts) Find the area enclosed by $y = 8 - x^2$, $y = x^2$, $x = -3$, and $x = 3$. (sketch the curves and the region of interest)

18. (5pts) Find the volume of the solid obtained by rotating about the y -axis the region bounded by $y = -2x^2 + 2x$ and $x = 0$.

19. (5pts) Find the volume of the solid obtained by rotating about the line $x = 2$ the region bounded by $y = x$ and $y = \sqrt{x}$.