Math 1160 Final Examination

May 4, 2012

NAME: ____________________________________________

Write neatly and show all your work in the space provided next to each question. Answers without work and illegible answers will not receive credit.

Each student is expected to follow Tulane’s Code of Academic Conduct. Any violation will be reported to the Honor Board for investigation.

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<th>Points Possible</th>
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1 (10 points)

An oak tree in City Park was struck by lightning during a thunderstorm in April. The tree is still standing upright, but needs to be supported by a straight metal rod. The rod will run from the top of the tree to a spot on the ground that is 11.7 feet from the trunk of the tree. The angle between the rod and the ground will be 16.5°. Find the length of the rod needed.
2  (10 points)

Verify the identity

\[
\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} = 2 \tan^2 x.
\]
3 (10 points)

Pictured below are two graphs. The top graph is the familiar graph of $y = \ln x$. Below that is the graph of $y = g(x)$, which has been obtained from the top graph by a combination of reflections and shifts. (No stretching has taken place.) During this process, the point $P$ on the top graph moves to the point $Q$ on the bottom graph. Each square on the grid is 1 unit by 1 unit.

1. (5 points) Describe a sequence of reflections and shifts that will produce the graph of $y = g(x)$. (Be sure to write them in the correct order.)

2. (5 points) Use your description in the first part to write an equation for $g(x)$. 
4 (10 points)

Evaluate the following limits.

1. (5 points) \( \lim_{\theta \to \frac{\pi}{6}} \frac{\tan(2\theta)}{\theta} \)

2. (5 points) \( \lim_{\theta \to 0} \frac{\tan(2\theta)}{\theta} \)
5  (10 points)

1. (5 points) Condense the expression

\[3 \ln(5) + \ln(x) - \frac{1}{2} \ln(y)\]

into the logarithm of a single quantity.

2. (5 points) Differentiate \(g(x) = \ln\left(\frac{e^x \sin x}{\cos x}\right)\). (Hint: Use properties of the logarithm and/or trigonometric identities to simplify before differentiating.)
6 (10 points)

Scientists introduce a total of 50 animals of an endangered species onto an island. The island is believed to have a carrying capacity of 1200. The size of the population can be modeled by the logistic model

\[ P(t) = \frac{1200}{1 + 23e^{-0.2t}}, \]

where the time \( t \) is measured in years.

1. (5 points) How many animals will there be after 10 years?

2. (5 points) When will there be 500 animals?
7  (10 points)

1. (8 points) Recall from Long Calculus I that an inflection point of the graph \( y = P(t) \) occurs when \( P''(t) = 0 \) and the concavity changes. The graph of a logistic model has a single inflection point. For the function

\[
P(t) = \frac{1200}{1 + 23e^{-0.2t}},
\]

find the value of \( t \) where \( P''(t) = 0 \). (You may continue your work onto the back of this sheet if necessary.)

2. (2 points) Which of the following statements is true about the time \( t \) found in Part 1? (Circle one.)

(a) The population is zero at this instant of time.
(b) The population takes a maximum value at this instant of time.
(c) The rate of change of the population is zero at this instant of time.
(d) The rate of change of the population takes a maximum value at this instant of time.
8 (10 points)

Find an equation of the tangent line to the curve $y = e^{\cos(3x)}$ at the point $\left(\frac{\pi}{6}, 1\right)$. 
9  (10 points)

1. (5 points) Write an algebraic expression for $\cos(\arctan x)$. (An algebraic expression is an expression in terms of $x$ that only involves the operations of addition, subtraction, multiplication, division, raising to a power, and taking a root. An algebraic expression may not contain a trigonometric function, exponential function, or logarithmic function.)

2. (5 points) Differentiate $f(x) = \cos(\arctan x)$. 
10  (10 points)

The displacement of a damped harmonic oscillator, in centimeters, is given by \( s(t) = 10e^{-\sqrt{3}t} \sin t \), where \( t \) is measured in seconds. What is the maximum displacement of the oscillator in the interval \( 0 \leq t \leq 2\pi \)? For this problem, you must work with radians (not degrees).
11 (10 points)
Evaluate the indefinite integral
\[ \int \frac{x^3 + x}{x^4 + 2x^2 + 1} \, dx. \]
Evaluate the indefinite integral
\[ \int e^x (1 + \cos(e^x)) \, dx. \]
13 (10 points)

Evaluate the indefinite integral
\[ \int_{\pi/4}^{7\pi/3} \sin^2 x \cos x \, dx. \]

Use the unit circle to give the exact value, not a decimal approximation.
14  (10 points)

Evaluate the definite integral

\[ \int_0^{\sqrt{3}} \frac{1}{4x^2 + 36} \, dx. \]
15  (10 points)

Determine which area is larger: (A) the area under the curve $y = \sin(x)$ and above the $x$-axis for $0 \leq x \leq \frac{\pi}{2}$; or (B) the area of a rectangle that has a horizontal length of $\frac{\pi}{2}$ and a vertical length of 1. Justify your answer.

Have a great summer!