MATH 1210 FINAL EXAM

TUESDAY, DECEMBER 7, 2010
8:00 AM - 12:00 NOON

Name: _____________________________ Section #: _____
Instructor: __________________________

INSTRUCTIONS:
This exam consists of both short and long answer questions. Short answer questions: No need to show your work. Write your answers in the boxes below. No partial credit. Long answer questions: No work - no credit.
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SHORT ANSWER QUESTIONS

1. Compute the limit.

(a) \[ \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right) \]

(b) \[ \lim_{x \to -\infty} \frac{5x^3 - 7x + 3}{4 - 3x^2 - 2x^3} \]

2. Let \( f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0, \\ 3 - x, & \text{if } 0 \leq x < 3, \\ (x - 3)^2, & \text{if } x > 3. \end{cases} \)

Evaluate

(a) \[ \lim_{x \to 0^+} f(x) = \]

(b) \[ \lim_{x \to 0^-} f(x) = \]

(c) \[ \lim_{x \to 0} f(x) = \]

(d) \[ \lim_{x \to 3^-} f(x) = \]

(e) \[ \lim_{x \to 3^+} f(x) = \]

(f) \[ f \text{ is discontinuous at } x = \]
3. Find $\frac{dy}{dx}$ for:

(a) $y = e^{2x} \cos(3x)$

(b) $y = \ln(\ln(\ln(x)))$

4. The equation of the tangent line to the graph of $y = x^2$ at $x = 1$ is:

5. If a snowball melts so that its surface area decreases at the rate of $1\text{cm}^2/\text{min}$, the rate at which the diameter decreases when the diameter is $10$ cm is:

Note: Surface area is given by $S = 4\pi r^2$, where $r$ stands for radius.
6. Given the data

\begin{align*}
&f(1) = 5 & f'(1) = 7 \\
g(1) = 3 & g'(1) = 0 \\
g(5) = 2 & g'(5) = 10
\end{align*}

compute the derivatives of each of the following at \( x = 1 \).

(a) \( f(x)g(x) \)

(b) \( \frac{f(x)}{g(x)} \)

(c) \( g^2(x) \)

(d) \( g \circ f(x) \)

7. If \( f''(x) = \sin x + \cos x \), \( f(0) = 3 \), \( f'(0) = 4 \), then \( f(x) \) is given by:

8. Find \( \frac{d}{dx} \int_0^x \frac{e^t}{1+t^2} \, dt \).
9. Let \( f(x) = x^2 + 1 \). Divide the interval \( 0 \leq x \leq 2 \) into four equal parts and estimate \( \int_0^2 f(x) \, dx \), correct to four decimal places, using

(a) the method of right end points

(b) the method of midpoints

10. Find the absolute maximum and the absolute minimum values of \( f(x) = 3x^2 - 12x + 5 \) over the interval \( 0 \leq x \leq 3 \).
LONG ANSWER QUESTIONS

11. Let \( f(x) = 5x^2 - 3x \). Find \( f'(x) \) using only the limit-definition of the derivative.

12. Use the quotient rule to show \( \frac{d}{dx} \tan x = \sec^2 x \).
13. Find the equation of the tangent line to the curve \( x^2 + 4xy + y^2 = 13 \) at the point (2, 1).

14. At what point on the curve \( y = (\ln(x + 4))^2 \) is the tangent line horizontal?
15. A particle is moving according a law of motion \( s = f(t) = t^3 - 12t^2 + 36t \) for \( t \geq 0 \). (\( t \) is measured in seconds and \( s \) in feet.)

(a) When is the particle at rest?

(b) What is the total distance traveled during the first 8 seconds?

16. Evaluate the limit (provide a justification).

\[
\lim_{x \to 0} \frac{\sin x - x}{x^3}
\]
17. (a) Find the intervals where the function $f(x) = x^3 - 9x^2 + 24x - 1$ is increasing and the intervals where it is decreasing.

(b) Does the function have an inflection point? If so, where?

18. Let $f$ be continuous on $[0, 1]$ and differentiable on $(0, 1)$. If $f(0) = 1$, and $f'(x) \leq 5$ for all $x$, what is the largest possible value for $f(1)$? *Hint:* Use the Mean Value Theorem.
19. A cylindrical can without a top is made to contain $V$ cm$^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

*Note*: Volume of the can = (area of the base) $\times$ height

20. Find the integrals:

(a) \[ \int_0^4 (x^2 + |x - 1|) \, dx \]

(b) \[ \int x^2 \sin(2x^3 + 1) \, dx \]
21. Sketch the region enclosed by the curves:

\[ y = x, \quad y = \frac{1}{x}, \quad x = 4 \]

(Take \( x \geq 0 \).) Find the area of this region.