Calculus II

Name:___________________

Instructor:______________

Final Exam

Spring 2012

Circle your answer below

(1) [a] [b] [c] [d] [e]  
(2) [a] [b] [c] [d] [e]  
(3) [a] [b] [c] [d] [e]  
(4) [a] [b] [c] [d] [e]  
(5) [a] [b] [c] [d] [e]  
(6) [a] [b] [c] [d] [e]  
(7) [a] [b] [c] [d] [e]  
(8) [a] [b] [c] [d] [e]  
(9) [a] [b] [c] [d] [e]  
(10) [a] [b] [c] [d] [e]  
(11) [a] [b] [c] [d] [e]  
(12) [a] [b] [c] [d] [e]  
(13) [a] [b] [c] [d] [e]  
(14) [a] [b] [c] [d] [e]  
(15) [a] [b] [c] [d] [e]  
(16) [a] [b] [c] [d] [e]  
(17) [a] [b] [c] [d] [e]  
(18) [a] [b] [c] [d] [e]  
(19) [a] [b] [c] [d] [e]  
(20) [a] [b] [c] [d] [e]  
(21) [a] [b] [c] [d] [e]  
(22) [a] [b] [c] [d] [e]  
(23) [a] [b] [c] [d] [e]  
(24) [a] [b] [c] [d] [e]  
(25) [a] [b] [c] [d] [e]  

Answers correct to 3 significant figure accuracy are to be chosen over “None of the above”
1) If $F(x) = \int_1^{\ln(x)} e^t \, dt$, for $x \geq 1$, then $F'(x) =$

- $[a] \ x$
- $[b] \frac{1}{\ln(x)}$
- $[c] \frac{1}{2\sqrt{\ln(x)}}$
- $[d] \ x\sqrt{\ln(x)}$
- $[e] \ None \ of \ the \ above$

2) Which substitution turns the integral $\int \sqrt{1 + \frac{3}{\sqrt{x}}} \, dx$ into the integral $\int 3\sqrt{u}(u - 1)^2 \, du$ ?

- $[a] \ u = \frac{3}{\sqrt{x}}$
- $[b] \ u = 1 - \frac{1}{\sqrt{x}}$
- $[c] \ u = x^3$
- $[d] \ u = 1 + \frac{3}{\sqrt{x}}$
- $[e] \ None \ of \ the \ above$

3) The area of the region between the graphs of $f(x) = x^2 - 4x + 10$ and $g(x) = 4x - x^2$ over the interval $[1,3]$ is equal to

- $[a] \ \frac{16}{3}$
- $[b] \ \frac{11}{2}$
- $[c] \ 5$
- $[d] \ \frac{19}{4}$
- $[e] \ None \ of \ the \ above$

4) The volume of the solid generated by revolving the region bounded by the curve $y = x^3$, the $y$-axis, and the line $y = 3$ about the $y$-axis is (approximately) equal to

- $[a] \ 10.23$
- $[b] \ 11.76$
- $[c] \ 12.42$
- $[d] \ 13.35$
- $[e] \ None \ of \ the \ above$

5) The volume of the solid generated by revolving the region in the first quadrant that is above the parabola $y = x^2$ and below the parabola $y = 2 - x^2$ about the $y$-axis is equal to

- $[a] \ \frac{\pi}{2}$
- $[b] \ \frac{\pi}{3}$
- $[c] \ \frac{\pi}{4}$
- $[d] \ \pi$
- $[e] \ None \ of \ the \ above$

6) If the natural length of a spring is 0.2 meter, and if it takes a force of 12 newtons to keep it extended 0.04 meter, then the work done in stretching the spring from its natural length to a length of 0.3 meter (in joules) is equal to

- $[a] \ 1.5$
- $[b] \ 1.6$
- $[c] \ 1.7$
- $[d] \ 1.8$
- $[e] \ None \ of \ the \ above$

7) Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and $f''$ is continuous. Then $\int_1^4 xf''(x) \, dx =$

- $[a] \ 1$
- $[b] \ 2$
- $[c] \ 3$
- $[d] \ 4$
- $[e] \ None \ of \ the \ above$
8) \( \int (\sin(x))^5 \, dx \) is a sum of terms each having the form \( c \cdot (\cos(x))^n \). The largest power \( n \) occurring in this sum is equal to

[a] 2  [b] 3  [c] 4  [d] 5  [e] None of the above

9) Which substitution should one choose to handle the integral \( \int \frac{\sqrt{x^2-4}}{x} \, dx \)?

[a] \( x = 2\sec(t) \)  [b] \( x = 2 \sin t \)  [c] \( x = \cos 2t \)  [d] \( x = 2 \tan 2t \)  [e] None of the above

10) Evaluating the integral \( \int \frac{3x^2-8x+13}{(x+3)(x-1)^2} \, dx \) leads to a function of the form \( 4 \ln|x+3| - \ln|x-1| - \frac{A}{x-1} + C \). The correct value for \( A \) is

[a] 1  [b] 2  [c] 3  [d] 4  [e] None of the above

11) Which method has been used to approximate the integral \( \int_a^b f(x) \, dx \), \( f \) being an arbitrary continuous function defined on the interval \([a, b]\), by means of the formula

\[
\frac{b-a}{12} \left[ f(a) + 4f \left( a + \frac{b-a}{4} \right) + 2f \left( a + \frac{b-a}{2} \right) + 4f \left( a + \frac{3(b-a)}{4} \right) + f(b) \right]
\]

[a] left endpoint approximation  [b] midpoint rule  [c] trapezoidal rule  
[d] Simpson’s rule  [e] None of the above

12) The improper integral \( \int_{-\infty}^{1} xe^{-x^2} \, dx \) converges, and its value is equal to

[a] \( -\frac{2}{e} \)  [b] 0  [c] \( -\frac{1}{2e} \)  [d] The integral does not converge  [e] None of the above

13) The length of the curve \( y = 5 \left( e^{\frac{x}{10}} + e^{-\frac{x}{10}} \right) \), for \(-10 \leq x \leq 10\) is approximately equal to


14) The area of the surface of revolution generated by revolving the curve \( y = \sqrt{x}, 0 \leq x \leq 4 \), about the x-axis is approximately equal to

[a] 36.18  [b] 37.32  [c] 38.15  [d] 39.62  [e] None of the above
15) If \( y \) is a solution of the differential equation \( \frac{dy}{dx} = \frac{x+3x^2}{y^2} \) such that \( y(0) = 6 \), then \( y(1) \) is approximately equal to

[a] 5.76  [b] 6.04  [c] 7.21  [d] 8.12  [e] None of the above

16) Which of the following is an integrating factor for the linear differential equation \( \frac{dy}{dx} + 3x^2y = 6x^2 \)?

[a] \( e^{-x^2} \)  [b] \( e^{3x^2} \)  [c] \( e^x \)  [d] \( e^{-x^2} \)  [e] None of the above

17) The differential equation \( \frac{dp}{dt} = 0.08p(1 - \frac{p^2}{1000}) \) is a logistic differential equation with carrying capacity \( M = \)

[a] 1000  [b] 80  [c] 0.00008  [d] This is not a logistic differential equation  [e] None of the above

18) The parametric curve given by the equations \( x = t^2 + 1 \) and \( y = t^3 - 4t \) has a horizontal tangent line in case \( t = \)

[a] \(-\frac{2}{\sqrt{3}}\)  [b] \(-2\)  [c] 2  [d] \(\sqrt{4}\)  [e] None of the above

19) The curve with polar equation \( r = 2 \cos \theta \) is a circle centered at

[a] \((-2,0)\)  [b] \((1,0)\)  [c] \((0,1)\)  [d] The curve is not a circle  [e] None of the above

20) The sequence \( a_1, a_2, \ldots \) where \( a_n = (1 + \frac{2}{n})^n \) converges, and its limit is equal to

[a] 7  [b] \(\sqrt{50}\)  [c] \(e^2\)  [d] The sequence does not converge  [e] None of the above

21) The series \( \sum_{n=1}^{\infty} \frac{1}{1+n^p} \) converges for \( p = \)

[a] \(e^{-1}\)  [b] \(\frac{1}{2}\sqrt{2}\)  [c] 0.7  [d] \(\frac{\pi}{3}\)  [e] None of the above
22) The series $\sum_{n=0}^{\infty} e^{-n-1}$ converges and its value is equal to

[a] $\frac{1}{e-1}$  [b] $\frac{e}{e-1}$  [c] $e^2$  [d] The series does not converge  [e] None of the above

23) The series $\sum_{n=1}^{\infty} (-1)^n f(n)$ converges in case $f(x) =$

[a] $\frac{2x+3}{x+5}$  [b] $\sqrt{x}$  [c] $\frac{\ln(x)}{x}$  [d] $\frac{\ln(x)}{1+\ln(x)}$  [e] None of the above

24) The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+5)^n}{7n+5}$ is equal to

[a] 5  [b] $\frac{5}{7}$  [c] $\frac{7}{5}$  [d] 7  [e] None of the above

25) Which of the following is a power series representation the function $f(x) = \frac{x^2}{(1-x)^2}$?

[a] $\sum_{n=0}^{\infty} nx^{2n}$  [b] $\sum_{n=0}^{\infty} (n+1)x^{2n}$  [c] $\sum_{n=0}^{\infty} (n+1)x^{n+2}$  [d] $\sum_{n=0}^{\infty} nx^{n+2}$  [e] None of the above