INSTRUCTIONS: You will be allowed 4 hours for this exam. It is closed book, closed notes, and you may use a calculator. Do the problems you know first. Indicate your answer clearly and show all your work. No work, no credit.

FOR INSTRUCTOR’S USE ONLY

1. _____  2. _____
3. _____  4. _____
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19. _____  20. _____
Problem 1. Find the equation of the plane containing the two lines

\[ \mathbf{r}_1(t) = (2 + 3t, -1, -1 - t), \quad \mathbf{r}_2(t) = (2 - t, -1 + 2t, -1). \]
Problem 2. Suppose a vector function $A(t)$ has magnitude $|A(t)| = 7$ for any $t$. Prove that the vector $A(t)$ and its derivative $A'(t)$ are perpendicular to each other, for all $t$. Assume $A'(t)$ exists at each $t$. 
**Problem 3.** If two vectors in space satisfy $\mathbf{v} \cdot \mathbf{w} = \sqrt{3}$ and and $\mathbf{v} \times \mathbf{w} = (1, 2, 2)$, then compute the angle between $\mathbf{v}$ and $\mathbf{w}$. 
Problem 4. Consider two triangles:

- the first triangle $\Delta_1$ is formed by two vectors $v$ and $2w$,
- the second triangle $\Delta_2$ is formed by $2v$ and $w + v$.

Do $\Delta_1$ and $\Delta_2$ have the same area? If yes, prove it. If no, give an example to disprove.
Problem 5. Start at the point \( P = (0, 0, 3) \) and move 10 meters along the curve

\[
x = 3 \sin t, \quad y = 4t, \quad z = 3 \cos t
\]

in the positive direction (direction of time increase). Where are you now, in space?
Problem 6. Find the local minima, maxima, and saddle points (if any) of the function

\[ f(x, y) = e^y(y^2 - 2x^2) \]
Problem 7. Consider the surface
\[ z = 2\sqrt{x^2 y} - 1. \]

(a) Find an equation for the tangent plane to the surface \( z \) at the point \( (x, y, z) = (1, 5, 4) \).

(b) Find the directional derivative of \( z \) in the direction of \( u = (-1, 2) \) at the point \( (x, y) = (1, 5) \).

(c) Find the direction that maximizes the directional derivative to \( z \) at the point \( (x, y) = (1, 5) \).
Problem 8. Evaluate the limit or show that it does not exist.

\[
\lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + 2y^2}.
\]
Problem 9. Suppose a function is defined as

\[ f(x, y, z) = xyz. \]

Use Lagrange Multiplies to determine the maximum possible value of \( f(x, y, z) \) under the constraint

\[ x^2 + y^2 + z = 1. \]

Assume \( x > 0 \) and \( y > 0 \).
Problem 10. Find $\frac{dz}{du}$ given the following:

\[ z = x^2 + xy \ln y \]
\[ x = 2u \]
\[ y = x^2 \]
Problem 11. Consider a double integral

$$\iint_D f(x,y) \, dA,$$

where $D$ is enclosed by $y = 2 - x^2$ and $y = x + 2$. Set up iterated integrals for BOTH orders of integration.
Problem 12. Evaluate the iterated integral

\[ \int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} e^{x^2+y^2} \, dy \, dx \]

by converting to polar coordinates.
Problem 13. Find the area of the part of the surface $z = \frac{1}{2}xy$ that lies above the sector $x^2 + y^2 \leq 4$ with $x \geq 0$ and $y \geq 0$. 
Problem 14. Evaluate the triple integral
\[
\iiint_E z \, dV,
\]
where \( E \) lies under the plane \( z = x + 2y + 1 \) and above the region in the \( xy \)-plane bounded by the curves \( x = y^2 \) and \( x = 1 \).
Problem 15. Use spherical coordinates to evaluate the integral

\[ \iiint_{E} \frac{x}{1 + x^2 + y^2 + z^2} \, dV, \]

where \( E \) is the half of the solid ball \( x^2 + y^2 + z^2 \leq 4 \) with \( x \geq 0 \).
Problem 16. Evaluate the double integral

\[ \iint_D \frac{2x - y}{(3x + y)^2} \, dA \]

by making an appropriate change of variables, where \( D \) is (the bounded region) enclosed by the lines \( 3x + y = 1, \ 3x + y = 4, \ 2x - y = 0, \) and \( 2x - y = 2. \)
Problem 17. Evaluate the line integral

\[ \int_C x^3 y \, ds \]

where \( C \) is the upper half of the circle \( x^2 + y^2 = 4 \) traversing from \((2, 0)\) to \((-2, 0)\).
Problem 18. Let $C$ be the unit circle $x^2 + y^2 = 1$ oriented positively (counterclockwise). Given the vector field

$$\mathbf{F}(x, y) = \begin{pmatrix} \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{y}{\sqrt{x^2 + y^2}} \end{pmatrix},$$

evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$
Problem 19. Let \( \mathbf{F}(x, y) = (4x^3y^2 - 2xy^3)\mathbf{i} + (2x^4y - 3x^2y^2 + 4y^3)\mathbf{j} \).

(a) Does there exist a potential \( f \) such that \( \mathbf{F} = \nabla f \)? If yes, find it. If no, explain why not?

(b) Compute the work done by the vector field \( \mathbf{F} \) in moving an object from \((0, 1)\) to \((1, 0)\).
Problem 20. Use Green’s Theorem to evaluate the integral

$$\int_C \sqrt{1 + x^3} \, dx + 2xy \, dy$$

where $C$ is the positively oriented triangle with vertices $(0,0)$, $(1,0)$, and $(1,3)$. 