NAME: ____________________________

MATH 224: FINAL EXAM
April 30, 2010
8:00 AM - NOON

Problem 1 (10 points). Find the solution for
\[ y' = \frac{x}{1+2y}, \quad y(-1) = 0. \]

Answer: ________________

Problem 2 (15 points). Find the solution of the initial value problem
\[ (2x+3)y' = y + (2x+3)^{1/2}, \quad y(-1) = 0. \]

Answer: ________________
Problem 3 (15 points). Solve:

\[
\frac{dy}{dx} = \frac{2 - \frac{y}{x}}{\ln x}.
\]

Answer:

Problem 4 (20 points). Find the general solution of the homogeneous equation

\[(x^2 + y^2)dx - 2xy dy = 0.\]

Answer:
Problem 5 (20 points). For the autonomous equation $y' = 6 + y - y^2$:

(a) Develop a phase line for the equation.

(b) Classify equilibrium points as stable/unstable.

(c) Sketch the equilibrium solutions on the $ty$-plane.
Problem 6 (15 points). A population of bacteria is growing exponentially. It doubles itself in 10 days. If there are 1000 bacteria present initially, how long will it take the population to reach 10,000?

Answer:

Problem 7 (25 points). \( \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 4 - t, \quad y(0) = -1, \quad y'(0) = 0 \)

(a) Find a solution of the associated homogeneous equation.
(b) Find a particular solution.
(c) Find the general solution.
(d) Find the solution satisfying the initial conditions.

Answer:

(a)

(b)

(c)

(d)
Problem 8 (15 points). Use the Laplace transform to solve
\[ y'' - 2y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 0. \]

Answer:

Problem 9 (15 points). Solve the system \( Ax = \bar{b} \), where
\[
A = \begin{pmatrix}
4 & 2 & -5 \\
-14 & -8 & 18 \\
-3 & -2 & 4
\end{pmatrix}, \quad \bar{b} = \begin{pmatrix}
-5 \\
16 \\
3
\end{pmatrix}.
\]

Answer:
Problem 10 (20 points). Find a basis for:

(a) Null space of $A = (\bar{v}_1, \bar{v}_2, \bar{v}_3)$.

(b) $\text{span}(\bar{v}_1, \bar{v}_2, \bar{v}_3)$, where

\[
\bar{v}_1 = \begin{pmatrix} -8 \\ 9 \\ -6 \end{pmatrix}, \quad \bar{v}_2 = \begin{pmatrix} -2 \\ 0 \\ 7 \end{pmatrix}, \quad \bar{v}_3 = \begin{pmatrix} 8 \\ -18 \\ 40 \end{pmatrix}.
\]

Answer:

(a)

(b)

Problem 11 (15 points). Find a fundamental set of solutions for $\ddot{y}' = Ay$, where

\[
A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -5 & -6 \\ -2 & 3 & 4 \end{pmatrix}.
\]

Answer:
**Problem 12** (15 points). The matrix $A$ has complex eigenvalues. Find a fundamental set of real solutions of the system

$$\gamma' = \begin{pmatrix} -1 & 1 \\ -5 & 5 \end{pmatrix} \gamma, \quad \gamma(0) = \begin{pmatrix} 1 \\ -5 \end{pmatrix}.$$

**Answer:**

**Problem 13** (15 points). The matrix $A$ has one real eigenvalue of multiplicity two. Find the solution of the system

$$\gamma' = \begin{pmatrix} -2 & -1 \\ 4 & 2 \end{pmatrix} \gamma, \quad \gamma(0) = \begin{pmatrix} 0 \\ -3 \end{pmatrix}.$$

**Answer:**