1. a) State the Mean Value Theorem.

b) Let \( f \) be _____ on \([a,b]\) and _____ on \((a,b)\). Then there exists at least one _____ such that

[Box]

10) A point \( z \) is a fixed point of \( f : \mathbb{R} \to \mathbb{R} \) if \( f(z) = z \).

Prove that if \( f'(x) < 1 \), for all \( x \), then \( f \) has at most one fixed point.
2. Complete the following definition:

a) Let $f: \mathbb{R} \to \mathbb{R}$. $f$ is uniformly continuous if for each __________ there is a ________ such that

b) $f$ is Lipschitz if there is a constant $M$ such that

$$|f(x) - f(y)| \leq M|x - y|.$$
3. Let $f : [a, b] \to \mathbb{R}$ be a bounded function.

Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of $[a, b]$.

(a) Define upper and lower sum

$U(f, P) =$

$L(f, P) =$

(b) $f$ is integrable over $[a, b]$ if $S = S$, where

$S =$

$S =$

(c) A useful theorem states that $f$ is integrable over $[a, b] \iff$ for each $\epsilon > 0$, there is a partition $P_\epsilon$ of $[a, b]$ such that
(a) If $f$ is __________ or __________, then $f$ is integrable.

(b) Let $f: [0,2]$ be bounded and continuous except at 1. Prove $f$ is integrable. You may assume a continuous function on $[c, d]$ is integrable. **Hint:** Use useful theorem in (c), take $\epsilon > 0$, "isolate" 1, and use $\delta/3$. 
4. a) Define: a function $f: \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at $a_0$ if

b) Prove from the definition that $f(x) = |x|$ is not differentiable at 0.
5. State the Fundamental Theorem of Calculus.

8. Let \( f : [a,b] \to \mathbb{R} \) be ______.

Then \( f \) has ______ and ______.

If \( f \) is any other ______, we also have ______.

6. Find the derivative of

\[ G(x) = \int_{x^3}^{4} \frac{t}{t^4 + 1} \, dt \]

\[ G'(x) = \]
6. A sequence \((x_n)\) of real numbers is a Cauchy sequence if, for every \(\epsilon > 0\), there is a \(N\) such that

\[
| x_n - x_m | < \epsilon \text{ for } n, m \geq N.
\]

7. \((x_n)\) converges if, for every \(\epsilon > 0\), there is a \(N\) such that

\[
| x_n - L | < \epsilon \text{ for } n \geq N.
\]

8. **Prove:** \((x_n)\) Cauchy \(\Rightarrow\) \((x_n)\) converges
The function $f$ pictured at right is continuous at $x_0$ (take my word for it).

I've drawn a pair of horizontal lines (i.e., chose $\delta > 0$).

Draw a pair of lines to show you know how to pick $\delta > 0$.

The function $f$ pictured at right is discontinuous at $x_0$. Choose $\delta > 0$ to show this.

$\varepsilon =$
1. Math 305 is the easiest math.

2. Common reasons:
   a) Dr. Hogan has a very direct.
   b) We feel duty after air change.
   c) We eat pizza instead of going to clean.
   d) All of the above.