1) a) Define what it means for a sequence of functions \( f_n(x) \) defined on an interval \( I \) to converge uniformly to a function \( f(x) \) on \( I \).

b) Give an example of a sequence, \( f_n(x) \), of continuous functions on \([0, 1]\) that converge to a continuous function \( f(x) \) for each \( x \in [0, 1] \) but do not converge uniformly. Prove your answer.

c) Show that if \( f_n(x) \) is a sequence of continuous functions on \([0, 1]\) which converge uniformly to a function \( f(x) \) on \([0, 1]\), then \( f(x) \) is continuous at each point \( x \).

2) a) Find the power series expansion for the function

\[
f(x) = \frac{x^3}{(1 - x)^2}
\]

about \( x = 0 \).

b) What is the radius of convergence of the power series in part a)?

c) Use part a) to compute \( f^{(20)}(0) \).

3) a) State the fundamental theorem of calculus.

b) Use part a) to compute \( F''(0) \) if

\[
F(x) = \int_0^x e^{-t^3} dt.
\]

c) If \( f_n(x) \) is a sequence of continuous functions which converge uniformly to a function \( f(x) \) on \([a, b]\), show that

\[
\lim_{n \to \infty} \int_a^b f_n(x) \, dx = \int_a^b f(x) \, dx.
\]

4) a) Define what it means for a set, \( F \), of functions to be equicontinuous on a set \( S \).

b) State the Arzela-Ascoli Theorem.

c) Suppose that \( f_n(x) \) is a uniformly bounded sequence of differentiable functions which have uniformly bounded derivatives on a set \([a, b]\). Show that a subsequence of the \( f_n \)'s converge uniformly on \([a, b]\) to a function \( f(x) \).