

## Homework VI: The Earth's Magnetic Field

### Introduction

The direction of the Earth's magnetic field at any point on the Earth can be described in terms of its magnetic azimuth or declination and its inclination.

The declination ( $\delta$ ) is measured positive ( $0^\circ$  to  $360^\circ$ ) clockwise from magnetic north. The inclination ( $i$ ) is measured positive downwards. Note that at the south magnetic pole the inclination is  $-90^\circ$ .

If the strength or magnitude of the magnetic field is given by  $H$ , then the horizontal component ( $H_h$ ) and the vertical component ( $H_v$ ) magnitudes can be expressed in terms of the inclination ( $i$ ) as:

$$H_h = H \cos (i)$$

and

$$H_v = H \sin (i)$$

If we assume that the Earth's magnetic field is represented by a dipole, then the horizontal component and vertical component of the field are as follows:

$$H_h = \frac{\mu_0 m \sin \theta_m}{4\pi r^3} \quad \text{and} \quad H_v = \frac{\mu_0 m \cos \theta_m}{2\pi r^3}$$

where  $\mu_0$  is the magnetic permeability of free space ( $4\pi \times 10^{-7}$  tesla m A<sup>-1</sup>) and  $m$  is the magnetic dipole moment ( $7.94 \times 10^{22}$  A m<sup>2</sup>) (A refers to the Ampere or Amp),  $r$  is the distance from the center of the Earth, and  $\theta_m$  is the magnetic co-latitude.

The angle of inclination for the magnetic dipole field is given by the relation

$$\tan (i) = H_h/H_v = 2 \cot \theta_m$$

Or, since  $\theta_m$  is the magnetic co-latitude  $= \pi/2 - \phi_m$ ,  $\tan i = 2 \tan \phi_m$ .

Because the strength of the magnetic field is defined as the perpendicular force ( $F_p$ ) acting on a charge ( $q_0$ ) with a velocity  $v$  within the field, the following conversion factors are relevant:

$$H = F_p / q_0 v \text{ (units: Newton/[Coulomb (meter/sec)]} = 1 \text{ tesla} = 10^4 \text{ Gauss} = 10^9 \text{ gamma)}$$

### Problems

1. The magnitude of the dipole field can be written in terms of the horizontal and vertical components as:

$$H = \sqrt{H_h^2 + H_v^2}$$

- a) show that the magnitude of the dipole field can also be written in terms of magnetic latitude  $\varphi_m$  ( $\varphi_m = \pi/2 - \theta_m$ )

$$H = \frac{\mu_0 m}{4\pi r^3} \sqrt{1 + 3\sin^2 \varphi_m}$$

- b) At what magnetic latitudes is the dipole field is the strongest and weakest?
- c) Calculate the maximum and minimum intensities of the Earth's dipole field.
- d) Assuming the dipole model, what is the maximum intensity of the magnetic field at the core-mantle boundary?
- e) Assuming the dipole model, at what distance above the Earth's surface is the magnitude of the field one half of its value on the surface at the magnetic pole?
2. Assume that the Earth's magnetic field is a dipole aligned along the geographic north-south axis. Keep in mind the following:

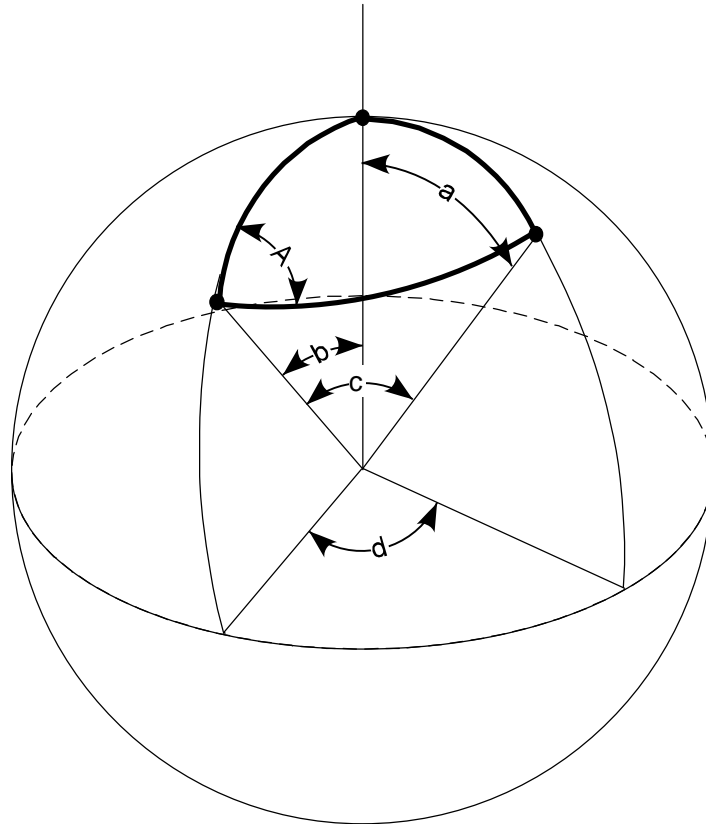
Magnetic colatitude ( $\theta_m$ ) is measured from  $0^\circ$  at the North pole to  $180^\circ$  at the South Pole. Longitude is measured positive in an eastward direction from the Greenwich Meridian (Longitude of London, England).

- a. What is the angle of inclination at London, U.K. located at  $51^\circ\text{N}$ ,  $0^\circ\text{E}$ ?
- b. What is the angle of inclination at Canberra, Australia, located at  $35^\circ\text{S}$ ,  $149^\circ\text{E}$ ?
- c. If the angle of inclination is  $49.1^\circ$  and you know that you are somewhere along the meridian  $90^\circ$  west of London, where are you?
- d. Magnetic measurements have been made on some lavas found at  $60^\circ\text{N}$ ,  $90^\circ\text{W}$ . The angle of inclination when these lavas were erupted is measured at  $37^\circ$ . At what magnetic latitude were these lavas erupted?

3. The poles of the Earth's magnetic field are located at  $73^\circ$  N,  $100^\circ$  W and  $68^\circ$  S,  $143^\circ$  E. The poles of the best fit dipole are located at  $79^\circ$  N,  $70^\circ$  W and  $79^\circ$  S,  $110^\circ$  E. Use the information concerning spherical trigonometry beginning on page 4 to answer the following questions.
- What is the surface distance between the geographic and magnetic poles?
  - What is the surface distance between the Earth's magnetic poles and the best fit dipole poles?
  - Using the best fit dipole model, determine the declination and inclination of the Earth's magnetic field at Boston  $\phi = 42.5^\circ$ ,  $\psi = -71^\circ$  ( $\psi$  is the Longitude, with west longitude measured as negative).
  - What is the intensity of the dipole field at Boston?
4. The measured declination and inclination of the paleomagnetic field in the Upper Triassic rocks located at  $41.5^\circ$  N and  $72.7^\circ$  W are  $\delta = 18^\circ$  and  $i = 12^\circ$ . Determine the position of the paleomagnetic pole.
5. The measured declination and inclination of the paleomagnetic field in Oligocene rocks located at  $51^\circ$  N and  $14.7^\circ$  E are  $\delta = 200^\circ$  and  $i = -63^\circ$ . Determine the position of the paleomagnetic pole.

## Spherical Triangles and Their Use in Solving Paleomagnetic Problems

We here consider some of the rules involved in solving problems that deal with spherical triangles. First we look at spherical triangles from a general point of view, and then look at how we would set up the spherical triangle to solve paleomagnetic problems.



For the spherical triangle shown above in heavy lines, the angles  $a$ ,  $b$ , and  $c$  are angles between lines radiating from the center of the sphere, and the angle  $A$  is one of the internal angles of the spherical triangle. Note that if we know three of the angles  $A$ ,  $a$ ,  $b$ , &  $c$ , we can solve for the other angle. The angle  $d$  is an angle measured in the equatorial plane. For such a spherical triangle, the following relations apply:

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A) \quad (1)$$

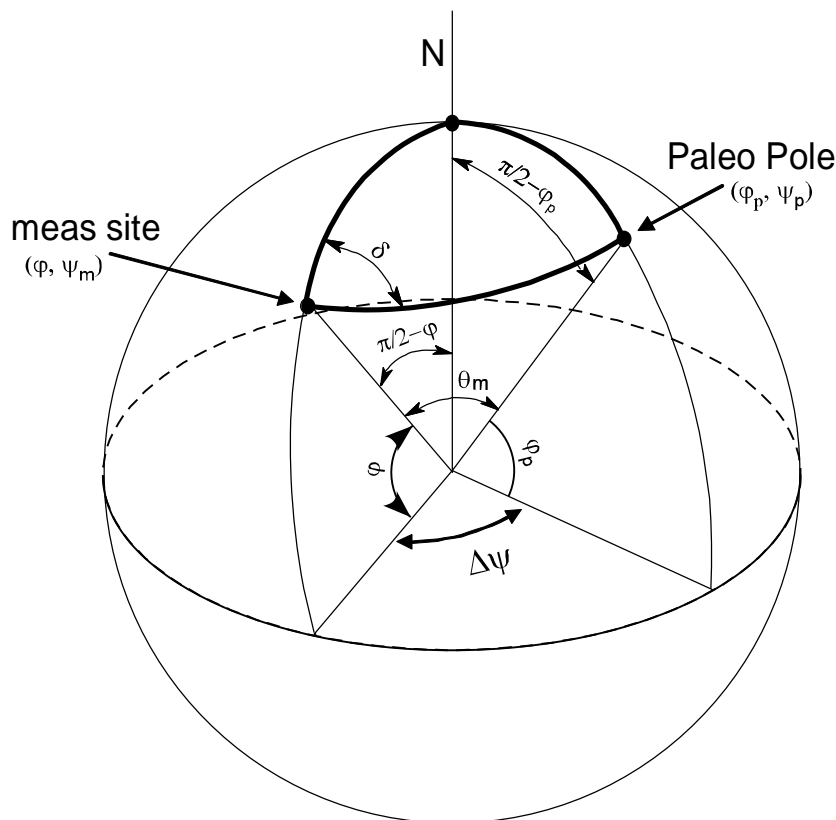
and, if  $\cos(c) > \cos(b)\cos(a)$  then:

$$\sin(d) = \sin(c)\sin(A)/\sin(a) \quad (2)$$

or, if  $\cos(c) < \cos(b)\cos(a)$  then:

$$\sin(\pi + d) = \sin(c)\sin(A)/\sin(a) \quad (3)$$

To set up a spherical triangle to solve paleomagnetic problems, we change the diagram as follows:



The angle A becomes  $\delta$ , the angle between geographic pole and the paleopole. This also corresponds to the measured paleomagnetic declination.

The angle a becomes the colatitude of the paleopole. The latitude of the paleopole is shown as the angle  $\varphi_p$ , so the colatitude is  $\pi/2 - \varphi_p$ .

The angle b becomes the colatitude of the site at which the measurement was taken. The latitude of the measurement site is shown as  $\varphi$ , so the colatitude of the measurement site is  $\pi/2 - \varphi$ .

The angle c becomes the magnetic colatitude of the paleopole,  $\theta_m$ . You can see this if you imagine rotating the spherical triangle upward so that the paleopole coincides with the geographic pole, then the angle between the measurement site and the paleopole would be the magnetic colatitude.

The geographic coordinates of the measurement site are latitude  $\varphi$ , longitude  $\Psi_m$ .

The geographic coordinates of the paleopole are latitude  $\varphi_p$ , longitude  $\Psi_p$ .

The geographic pole is at latitude  $90^\circ$ .

Remember that longitude is measured eastward of the Greenwich Meridian as positive, and westward as negative, and colatitude and magnetic colatitude are measured from the north

geographic or north magnetic pole at  $0^\circ$  to the south geographic or magnetic pole at  $180^\circ$ .

The angle  $d$  becomes the difference in longitude between the measurement point and the paleopole,  $\Delta\psi = \Psi_m - \Psi_p$

Now that the triangle is set up, we can rewrite the rules for spherical triangles as follows:

Equation (1), above becomes:

$$\cos(\pi/2 - \varphi_p) = \cos(\pi/2 - \varphi)\cos(\theta_m) + \sin(\pi/2 - \varphi)\sin(\theta_m)\cos(\delta) \quad (4)$$

Noting that  $\sin(x) = \cos(\pi/2 - x)$ , we can also write equation (4) as:

$$\sin(\varphi_p) = \sin(\varphi)\cos(\theta_m) + \cos(\varphi)\sin(\theta_m)\cos(\delta) \quad (4a)$$

If  $\cos(\theta_m) > \cos(\pi/2 - \varphi_p)\cos(\pi/2 - \varphi)$  then equation (2) above becomes:

$$\sin(\Delta\psi) = \sin(\theta_m)\sin(\delta)/\sin(\pi/2 - \varphi_p) \quad (5)$$

or, using the exchange of sines and cosines, as noted above,

$$\sin(\Delta\psi) = \sin(\theta_m)\sin(\delta)/\cos(\varphi_p) \quad (5a)$$

And, if  $\cos(\theta_m) < \cos(\pi/2 - \varphi_p)\cos(\pi/2 - \varphi)$  then equation (3) above becomes:

$$\sin(\pi + \Delta\psi) = \sin(\theta_m)\sin(\delta)/\sin(\pi/2 - \varphi_p) \quad (6)$$

or alternatively

$$\cos(\Delta\psi) = \sin(\theta_m)\sin(\delta)/\cos(\varphi_p) \quad (6a)$$

Finally, if we need to measure the surface distance between any two points on the surface of the sphere, the length of the arc,  $L$ , in the diagram below is given by:

$L = r\alpha$ , where the angle,  $\alpha$ , is measured in radians, and  $r$  is the radius of the Earth.

