THE ROLE OF LARGE DEFORMATIONS IN TRABECULAR BONE MECHANICAL BEHAVIOR

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INTRODUCTION

Trabecular bone exhibits nonlinear stress-strain behavior even at apparent strains below the yield point (<0.5%) [1]. Two factors contribute to this behavior: 1) material nonlinearity and 2) geometrical nonlinearity (i.e. large deformations). Due to inter-anatomic site variations in trabecular architecture and volume fractions [2], contribution of these factors are likely to be site dependent. For example, experimentally observed geometrical nonlinearities such as bending and buckling [3] should have a more important role for low density vertebral trabecular bone than it does for the much denser femoral neck trabecular bone.

The effects of large deformations can be important in determining localized stress-strain fields which are important in the investigation of mechanically induced bone adaptation and remodeling [4]. To date, high-resolution finite element models have been used to investigate tissue [5] and apparent level elastic properties [6], as well as the strength [7] of trabecular bone. While these studies provide insight into trabecular bone mechanical properties and their relations to architecture and volume fractions, only geometrically linear finite element analysis have been used. To the best of our knowledge, the effects of geometrical nonlinearities have not been included in µCT based finite element models of trabecular bone.

The goal of this study was to investigate the effects of geometrical nonlinearities, i.e. large deformations, on the elastic behavior of trabecular bone. Specifically, our objectives were to 1) compare tensile and compressive apparent stresses at 0.5% strain obtained through geometrically nonlinear analyses against those of linear analyses for four human anatomic sites, and 2) for a vertebral bone specimen, compare differences in the tissue-level stress distributions between geometrically nonlinear vs. linear analyses.

METHODS

Four human trabecular bone cylindrical specimens from four anatomic sites (Table 1) with 8-mm diameter and 13-mm average length were used. Specimens were chosen such that their volume fractions were close to the reported mean values for each anatomic site [1]. Specimens were scanned using µCT (Scanco Medical AG, Bassersdorf, Switzerland) at a resolution of 22 µm. High-resolution finite element models of entire cylindrical specimens were created after the resolution was coarsened to 44 µm for low density VB, GT, PT specimens and 66µm for the high-density FN specimen, to save computation time (Table 1).

Three uniaxial stress simulations up to 0.5% strain were performed for each specimen: linear elastic, and geometrically nonlinear in tension and compression. For the geometrically nonlinear analyses, the trabecular tissue was modeled as an isotropic St.Venant-Kirchhoff material:

$$\boldsymbol{S} = \boldsymbol{D}\,\boldsymbol{E} \qquad (1)$$

where $\boldsymbol{S}$ is the second Piola-Kirchhoff stress, $\boldsymbol{E}$ is the Lagrangian strain, and $\boldsymbol{D}$ is the elasticity tensor, which has only two material constants (E, ν) for an isotropic linear material. In all models, finite elements were assigned a Young's modulus (E) of 1 GPa and a Poisson's ratio (ν) of 0.3. A custom parallel finite element implementation based on the research code FEAP (University of California, Berkeley) with a parallel mesh partitioner ParMetis (University of Minnesota), PETSc (Argonne National Labs) and a parallel multigrid solver [8] was used for all analyses. All 12 analyses were performed on 32 processors of an IBM SP3 parallel supercomputer and in total required 1149 CPU hours. Linear, geometrically nonlinear in tension and compression required 3, 140, and 150 CPU hours on average, respectively.

Table 1. Specimen and model data

Anatomic Site	Volume Fraction	Element Size (µm)	# of elements
Vertebral Body - VB	0.089	44	908,651
Greater Trochanter - GT	0.104	44	927,992
Proximal Tibia - PT	0.114	44	873,284
Femoral Neck - FN	0.269	66	516,505

Stress-strain curves were obtained for all models and the percent difference in apparent stresses was calculated between nonlinear and linear curves ($\Delta\sigma/\sigma_{linear}$) at 0.5% strain.

For the vertebral bone specimen (VB), maximum principal stresses for each element were calculated. The ratio of tissue subjected to extreme tensile and compressive loading was calculated to determine the relative amount of bending in trabeculae. R_T, was defined as the number of elements exceeding 5 MPa stress to the number of elements exceeding –5 MPa for apparent tensile loading and was calculated for both linear and nonlinear analyses. Similarly, R_C, was calculated for compressive loading.

RESULTS

The stress-strain curves exhibited the same trend for all anatomic sites in which nonlinear tension resulted in stiffening and nonlinear compression resulted in softening compared to the linear case (Fig. 1). With increasing volume fraction, the difference in stresses between nonlinear vs. linear solutions at 0.5% apparent strain decreased (Fig. 1). Mean tissue stresses, and the ratios R_T and R_C are given in Table 2.

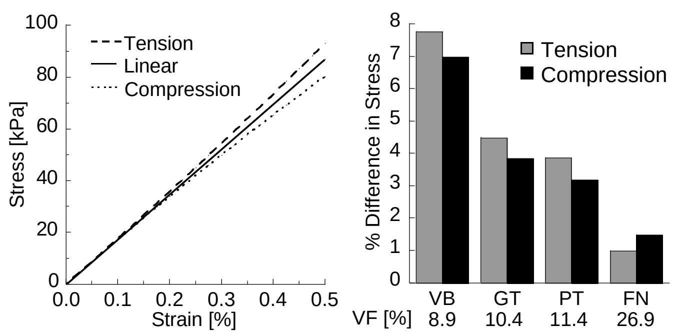

Figure 1. Left: Stress-strain curves for VB specimen showing differences in tensile and compressive behavior. Similar behavior was observed for all four sites. **Right:** Differences in stresses between nonlinear vs. linear solutions ($\Delta\sigma/\sigma_{linear}$) at 0.5% strain decreased with increasing volume fraction (VF). Note the underestimation in compression is also shown as positive.

Table 2. Element stresses for VB specimen

	Tension		Compression	
	Mean Stress ± SD	R_T	**Mean Stress ± SD**	R_C
Linear	0.98 ±1.79	13.0	-0.98 ±1.79	13.0
Nonlinear	1.06 ± 1.83	16.4	-0.90 ± 1.75	10.6

DISCUSSION

The inclusion of geometrically nonlinear deformations in μFE analyses of trabecular bone resulted in nonlinearities in tensile and compressive apparent stress-strain behavior at strain levels (≤ 0.5%) below the apparent yield point. For all anatomic sites, geometrically nonlinear analyses over- and under-estimated apparent stresses in tension and compression, respectively, when compared to linear analyses. These results indicate that geometrically nonlinear deformations do play a role in the mechanical behavior of low-density trabecular bone, even at relatively small strains.

Investigation of the maximum principal stress distributions for the human vertebral bone specimen indicated that while mean stresses were less than 8% different for linear and nonlinear analyses, the ratios R_T and R_C were as much as 26% different than the linear values for each loading mode. The differences in these ratios reflect the fact that the geometrically nonlinear analyses result in differences in tissue level stress distributions in tension and compression loading, while linear analyses result in the same distributions for both loading modes. For the nonlinear analyses, $R_C < R_T$ indicates that more bending is present when apparent loading is compressive. This results in a reduction of stiffness of the slightly oblique trabeculae (with respect to the loading axis) as the structure deforms under compression providing an explanation for the softening (Fig. 1). In tension straightening of the slightly oblique trabeculae would cause stiffening, which is consistent with our observations.

While experimental data for stress-strain curves up to 0.5% strain indicate a concave down trend for all anatomic sites both in tension and compression [1], our geometrically nonlinear analyses exhibited such behavior only in compression. This discrepancy in tension is most likely due to the exclusion of tissue constitutive nonlinearity in our analyses. Trabecular tissue has strong tensile-compressive yield strength asymmetry (~0.5), with a tensile yield strain of only 0.41%. [9]. Even at small apparent strains, tissue exceeding this tensile yield strain will cause a reduction in stiffness of the trabeculae, which will also reduce the apparent stiffness. These findings indicate that more realistic behavior is to be expected by combining geometrical nonlinearity with material nonlinearity and that both may be equally important in overall failure mechanisms [10].

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