"ENGINEERING" MECHANICS OF ADHESION OF BIO-CAPSULE AND BIO-MEMBRANE TO A PLANAR SUBSTRATE

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Abstract

A new theoretical model is presented for the adhesion of a thin-wall capsule onto a planar substrate based on linear elasticity and a simple energy balance. Adhesion of a bio-membrane clamped at the perimeter onto a substrate is also formulated as "punch test". The new method can be used to investigate thin film adhesion in the presence of specific / non-specific surface forces and chemical environment.

Introduction

Bio-adhesion is an important in many aspects such as metastasis, tissue assembly and drug delivery capsules. In the presence of attractive surface forces, bio-capsule (e.g. liposome) or bio-membrane is drawn to a substrate, deforms elastically until equilibrium is reached. In this paper, we show how "engineering mechanics" is applied to model the adhesion and delamination processes of (i) spherical micro-capsule adhered onto a planar substrate in the presence of osmosis [1], and (ii) a new axisymmetric "punch test" for in-situ adhesion measurement for ultra-thin bio-membrane [2-3].

Mechanics of capsule adhesion

A few assumptions are needed for modeling: (i) capsule wall allows only film stretching; (ii) enclosing liquid is *incompressible*; (iii) adhesion forces are effective only upon direct contact; (iv) visco-elasticity, thermal undulations and the surface charge upregulation are ignored. All lengths hereafter are measured in units of original radius, R_0. The constant volume is $V_0 = 4\pi/3$, and initial surface area $A_0 = 4\pi$. Upon contact with a substrate, the capsule deforms to a truncated sphere with a radius, R, contact radius, a, and $(a/R) = \alpha = \sin\theta$ (Figure 1). Thus, volume becomes

$$V = \frac{2\pi R^3}{3}\left[1+\left(1+\frac{\alpha^2}{2}\right)(1-\alpha^2)^{1/2}\right] = V_0 \qquad (1)$$

and the enlarged surface area (excluding the contact circle),

$$A = 2\pi R^2[1+(1-\alpha^2)^{1/2}] \qquad (2)$$

The uniform biaxial membrane strain is roughly

$$\varepsilon = (1/2)\ [A/(A_0 - \pi a^2) - 1] \qquad (3)$$

corresponding to a membrane stress of $\sigma = C\ \varepsilon$, with C the extensional rigidity (i.e. $Eh/(1–\nu)$ in a linear membrane), h the film thickness, E and ν the elastic modulus and Poisson's ratio respectively. A capsule initially adhered onto a substrate is now exposed to a dilute solution, giving rise to an internal osmotic pressure and a liquid influx of volume, V_{in}. The resulting volume becomes $V = (1+ \nu)$ and radius $R = (1+ \nu)^{1/3}$ with $\nu = V_{in}/V_0$. A strain energy release rate, G, defined to be

$$G = (1-\cos\theta)\ C\varepsilon + C\varepsilon^2 \qquad (4)$$

such that equilibrium is reached when $G = W$, the interfacial adhesion energy. Figure 1 shows a/R as a function of ν. In an isotonic solution, the capsule experiences zero osmotic pressure and the contact circle depends only on the adhesion strength. Osmosis leads to a trajectory **ABCD** for $w = (W/C) = 10^{-3}$. At point **A**, the capsule achieves a maximum contact radius at specific adhesion strength. At point **D**, the capsule pinches off the substrate with zero contact. Our new theory agrees favorably with experimental measurement for a variety of liposomes.

Adhesion measurement using a "punch test"

To facilitate holding a thin fragile bio-membrane in place, it is clamped by two rings with radius of central opening, *a*. A cylindrical punch of radius, *a*, is adhered to the film via the opening before an external force, *F*, is exerted to drive a delimination through the adhered interface (Figure 2). Deformation of the film is governed by

$$\kappa\ \Delta^2 w - \sigma h\ \Delta w = F\ \delta(r) \tag{5}$$

with $\kappa = Eh^3 / 12(1-\nu^2)$ the flexural rigidity, and $\delta(r)$ the delta function. Assuming an average uniform membrane stress on the diaphragm and satisfying the boundary conditions ($w(r=c) = w_0$, $w(r=0) = 0$, $w'(r=\mathrm{c}) = 0$, and $w'(r=a) = 0$, where c the radius of contact circle), the differential equation was solved analytically yielding the film profile $w(r)$ and mechanical response without delamination $F(w_0)$. For delamination to occur along the film-punch interface, the strain energy release rate, *G*, is derived from a simple energy balance and the corresponding mechanical response with delamination, $F(w_0)$. The stability of the mechanical system is considered, and it can be shown easily that when $(\mathrm{d}w_0/\mathrm{d}F) = 0$, the delamination will propagate spontaneously until the punch is completely separated from the film, or a "pull-off". From linear elasticity, it was shown earlier that the when $(c/a) \approx 0.18$, "pull-off" occurs. The corresponding critical force and punch displacement are also obtained. The analysis was lately extended to pre-stressed films. "Pull-off" occurs within the range of $0.18 = (c/a) = 0.37$, depending on the magnitude of intrinsic membrane stress.

The new method allows adhesion measurement of ultra thin film. When ligand-receptor pairs are deposited on the punch and film, specific adhesion forces can also be measured. By varying the punch and hole dimension (*a*), the density of the molecular pair can also be measured. One advantage of the new geometry is that measurement of either F or w_0, and c/a at "pull-off" is sufficient to determine the adhesion energy and intrinsic membrane stress.

Conclusion

We have shown how solid mechanics is applied to construct a theoretical mechanical model for the adhesive contact mechanics of a capsule onto a substrate in the presence of osmosis. Rise in osmotic pressure within the capsule leads to shrinkage in contact area and even lift-off from the substrate. The axisymmetric punch test can be used to quantify the adhesion of an ultra-thin bio-membrane by measuring the critical load, punch displacement and contact radius at "pull-off".

References

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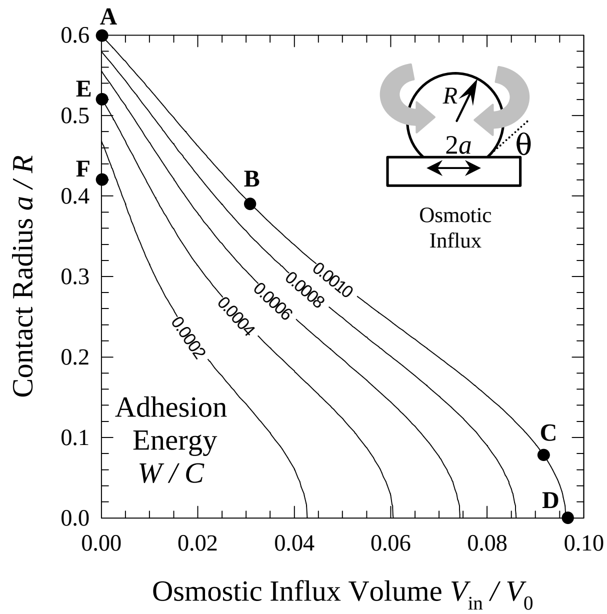

Figure 1. Contact radius as a function of osmotic influx volume. Inset shows the osmotic pressure buildup.

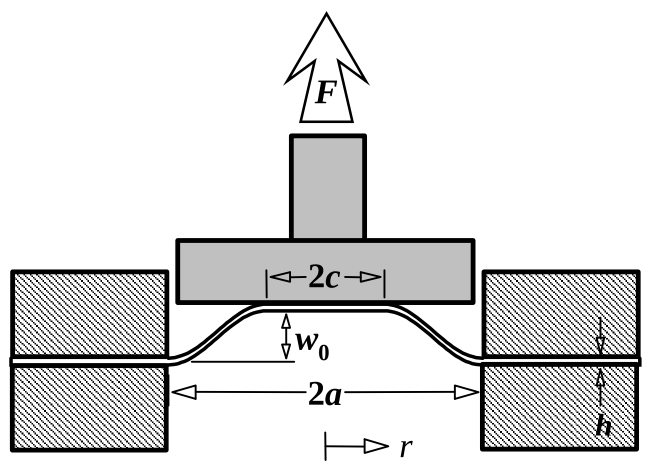

Figure 2. Schematic of axisymmetric punch test. The membrane is clamped by two identical rings with a central opening of radius *a*.