

# SCALE-INDEPENDENT BIOMECHANICAL OPTIMIZATION

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## INTRODUCTION

In this paper we present a scale-independent method of optimization with a stochastic global optimization approach introduced by Kennedy and Eberhart [1], the Particle Swarm Optimizer (PSO). We apply this method to the biomechanical system identification problem of finding positions and orientations of joint axes in body segments through the processing of experimental movement data [2,3]. We compare its performance to the BFGS optimizer which falls under a class of optimizers more commonly used for this application.

Traditionally, gradient-based methods such as the BFGS algorithm have been used to solve joint parameter identification problems, but major drawbacks to these methods are their sensitivity to problem scaling and algorithm parameter selection. These drawbacks require a costly and time-consuming parameter sensitivity studies to be carried out for a problem before consistently acceptable results can be obtained. In addition, the presence of noise in the data will often cause premature convergence to an incorrect solution.

The PSO method has some very desirable qualities that can be exploited in these types of problems. First, because it requires no gradient evaluations and because of the way it is formulated, the algorithm is insensitive to scaling of the design variables. Second, because of the algorithm's simplicity, there are very few parameters to tune, and even these have been shown to be relatively problem independent. Finally, the concurrent nature of the swarm algorithm lends it to parallelization, enabling the solution of problems that are too computationally challenging for single-processor machines. The need for greater computational power is common in the search for more realistic and accurate engineering models [4], which currently can only be addressed by the use of parallel algorithms.

## METHODOLOGY

For our test problem, we have elected to use a 2 degree-of-freedom (DOF), 3-dimensional ankle kinematic model requiring 12 parameters [2]. This model is used to generate synthetic trajectories of markers fixed to the foot and shank (three per segment). An optimization approach was then followed to recover the original joint

parameters from this synthetic marker trajectory data. One of the primary reasons for this approach is that any solution found by either algorithm can be quantified in terms of final design variable errors. Therefore, we are able to make an immediate evaluation of the performance of the optimization algorithm that was used. In addition, we have control over the magnitude and other characteristics of any numerical noise we introduce into the system to emulate experimental measurement errors, and we are able to observe the impact varying any of these has on the optimization algorithm.

The unconstrained optimization (or system identification) problem can be stated as follows:

$$\min_x f(x) = \sum_t^n e(x,t) \quad (1)$$

with

$$e(x,t) = \min_p \sum_i^m \sum_j^3 (m_j(t) - c_j(x,p))^2 \quad (2)$$

where  $x$  is a proposed joint position and orientation in the body segment, which will have a corresponding marker configuration  $c(x)$ . The fitness (Eq. 1) of this marker configuration is evaluated according to how closely it can be aligned (in three dimensions  $j$ ) to all of the  $m$  markers in configuration  $m(t)$  over all  $n$  recorded time frames  $t$ . This matching or alignment of proposed virtual to observed marker locations is done in a separate optimization step (Eq. 2) by means of a non-linear least squares fit, where  $p$  is an alignment operator [5].

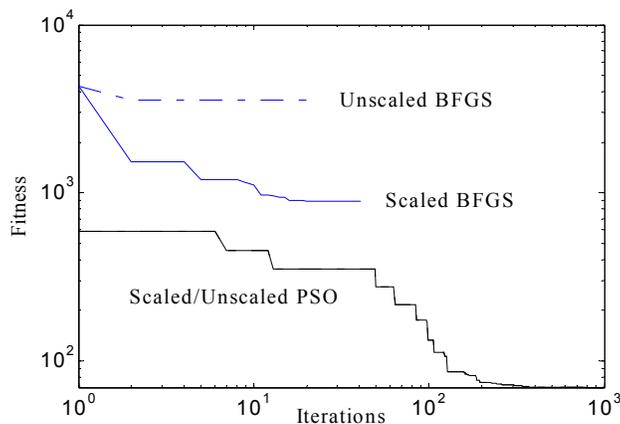
In order to demonstrate the potential sensitivity to scaling in both PSO and BFGS methods, the ankle joint identification problem was first defined using the original units of cm and radians for the location and orientation design variables respectively. Bounds on these design variables were chosen to enclose a physically realistic interval around the solution point in the design space.

The scaled version of this problem was then obtained by normalizing all 12 variables to be bounded within  $[-1,1]$ .

Numerical noise was introduced by means of superimposing a sine wave with a random period, phase, and amplitude (limited to a maximum of 1 cm) onto the marker data. This was done to emulate artifacts caused by skin and soft tissue movement and by camera resolution limitations found in real data.

In generating the synthetic marker trajectory data, we endeavored to simulate real life data which are analyzed in exactly the same manner. As such, the problem, while still being analytical, required the use of parallel processing due to the sheer amount of data to be processed. Both the PSO and the unconstrained BFGS gradient-based method were parallelized and evaluated on a cluster of 29 Linux based PCs in the UF HCS Research Laboratory (1.33 GHz Athlons with 256MB memory on a 100Mbps switched Fast Ethernet network).

## RESULTS



**Figure 1: Fitness history for typical BFGS and PSO runs**

Before using the parallelized VisualDOC (Vanderplaats R & D, Colorado Spring, CO) implementation of the BFGS algorithm, a parameter sensitivity study was performed in order to obtain the optimum initial forward finite difference and termination parameters of  $10^{-2}$  and  $10^{-5}$  respectively. For the parallel PSO, the standard general recommended parameters were used [6].

A total of 10 unscaled and 10 scaled (or normalized) optimization runs were performed for both the PSO and BFGS algorithms (Table 1). For both methods, the optimizations were started at randomly chosen points within the bounds. The same starting point locations were used throughout all of the runs when switching to the scaled problem in order to obtain a fair comparison for both algorithms.

The BFGS algorithm converged prematurely in all 10 unscaled runs, never obtaining even approximately correct joint positions and orientations, resulting in poor fitness values (large cumulative marker errors). Restarting the algorithm at the termination points yielded no improvement. After scaling the problem, however, we obtained convergence in 7 out of the 10 runs to an approximately correct solution. Again, all attempts to further improve these by a restart of the algorithm are unsuccessful.

In contrast, with the PSO method, both unscaled and scaled optimizations produced a final fitness errors on the order of the noise level with very little standard deviation as compared to the BFGS results. However, this reliability and accuracy came at a high cost in terms of function evaluations (see Table 1).

When comparing fitness value drift between the scaled and unscaled problems during the optimization (Figure 1), we observed

only a minute difference for the PSO algorithm, on the order of  $10^{-4}$ , due to numerical round-off and truncation. In contrast, the scaled BFGS solution very quickly diverged from the unscaled solution and in all but two cases terminated at an entirely different result than found for the unscaled problem. In these two cases, both the scaled and unscaled optimizations failed to converge to the approximate solution.

	Algorithm	Unscaled	Scaled
Mean fitness error	BFGS	$3806 \pm 2477$	$889 \pm 1601$
	PSO	$69.28 \pm 4$	$69.31 \pm 2$
Mean function evals	BFGS	65	305
	PSO	19700	20040

**Table 1: Comparison of unscaled and scaled optimization results for BFGS and PSO algorithms**

## CONCLUSIONS

The BFGS requires scaling of the problem and an extensive parameter sensitivity study before it is able to find the general solution region consistently. Even with both these measures, it still tends to become trapped in local minima as can be seen from the large standard deviation values in Table 1. The main advantage of this method is its efficiency in terms of function evaluations. In contrast, the PSO is very reliable in finding the correct solution region, and is insensitive to both the scaling of the problem and initial algorithm parameter selection. Its main drawback is the high cost in terms of function evaluations because of slow convergence in the final stages of the optimization. A hybrid approach could be advantageous if an efficient transition criterion could be found and optimum parameters for the BFGS algorithm are known in advance.

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