AN IMPROVED ELASTIC MODEL FOR INDENTATION OF A MICROCAPSULE BY AN ATOMIC FORCE MICROSCOPE TIP

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Abstract

A new elastic model is presented for the mechanical response of a thin-walled bio-mimetic microcapsule upon indentation by an atomic force microscope (AFM). The new theory is rigorously compared to two existing, which turn out to be the two limiting cases of our general solution. The constitutive relation is linear at small indentation depth and gradually turns cubic.

Introduction

Bio-mimetic microcapsules are important in novel drug delivery systems. The thin elastic capsule wall plays an important role in the mechanical behavior (e.g. adhesion to a substrate or apposing capsules) when subjected to external stimuli such as mechanical forces and internal osmotic pressure. Mechanical characterization of these capsules provides critical information for design parameters. Existing characterization methods include: (i) compresion of a single cell, (ii) profiling using the optical interference of the contact interface between a capsule and a glass substrate, (iii) indentation using an AFM probe. In this paper, we will focus on constructing an elastic model for the AFM indentation (Figure 1) and will compare with existing theories by Boulbitch [1] and Yao [2].

Theory

When a spherical capsule of radius, R, wall thickness, h, elastic modulus, E, Poisson's ratio, ν, turgor pressure, p, original membrane pressure, $\sigma_0 = pR/2h$, are the elastic modulus and Poisson's ratio is indented by an external force, F, via an AFM tip of radius, c, the process can be understood as a 2 step process: (i) the capsule is compressed by 2 parallel plates forming 2 planar contact circles at the polar regions with radius, a; (ii) one plate is then replaced by the AFM tip, resulting in a dimple of radius, a. The dimple profile can be found by linear elasticity [3,4]

$$\kappa\ \Delta^2 w - \sigma h\ \Delta w = -p + F\ \delta(r) \tag{1}$$

with $\kappa = Eh^3 / 12(1-\nu^2)$ the flexural rigidity, σ the apparent membrane stress within the dimple, and $\delta(r)$ the delta function. A few assumptions are taken: (i) the dominant deformation mode is membrane stretching of the wall with bending moment a perturbation; (ii) the deformation is assumed small and local such that increase in p and R are negligible; (iii) equilibrium requires $F = (\pi a^2)\ p$; and (iv) σ is uniform within the dimple. The two collapsed polar caps have an original height of $w_c \approx a^2/2R = F / 2\pi pR$. The external force is supported by the local membrane stress and bending moment within the dimple. The external load induces a local concomitant stress σ_m superimposing on the intrinsic membrane stress, such that $\sigma = \sigma_0 + \sigma_m$, where

$$\sigma_m = \left[\frac{E}{2(1-\nu^2)}\right] \int_c^a \frac{1}{2}\left(\frac{dw}{dr}\right)^2 r\,dr \Bigg/ \int_c^a r\,dr \tag{2}$$

The global geometry of a truncated sphere resumes beyond the dimple ($r > a$). The dimple profile is found to be

$$\omega = \frac{1}{\beta}\{C_1[I_0(\beta\xi) - I_0(\beta)] - C_2[K_0(\beta\xi) - K_0(\beta)]\} - \frac{\varphi}{\beta^2}\log\xi - \frac{\rho}{2\beta^2}(1-\xi^2) \tag{3}$$

with I_i and K_i the i^{th} order of the first and second kind modified Bessel functions respectively, and C_1 and C_2 some functions of φ, $\xi=r/a$, $\zeta=c/a$, $\omega=w/h$, $r=\rho a^4/2\kappa h$, $\varphi=Fa^2/2p\kappa h$, $\beta=(\sigma h a^2/\kappa)^{1/2}$ and $\beta^2=\beta_0^2+\beta_m^2$. The dimple depth is given by $\omega_0=\omega(\xi=0)$. It can be shown that $\beta_m^2 = 6\,[f(\beta)-f(\beta\xi)]\,/\,(1-\zeta^2)\beta^2$ with $f(x)$ some function given earlier [3]. The relation $\varphi(\omega_0)$ can now be found analytically by eliminating β_m. In case of pure stretching (κ=0), the constitutive relation $F(w_0)$ is shown in Figure 2, which is linear at small w_0 and cubic otherwise. In the linear region, the total displacement traveled by the AFM tip is given by

$$w_t = \frac{F}{2\pi pR}\left[\log\left(\frac{a^2}{c^2}\right)+\frac{c^2+a^2}{a^2}\right] \tag{4}$$

which is linear ($\varphi \propto \omega_0$).

Discussion

It is interesting to compare our new model with the existing theories. Boulbitch [1] assumes a dominant bending moment with negligible stretching, and finds a different dimple profile, $w = w_t\, K_0(\xi)/K_0(\zeta)$ and a linear spring constant $k = \pi R p\zeta\,[K_1(\zeta)/K_0(\zeta)]$. Comparing with the present model, the following terms are ignored in Boulbitch's equations: (i) $I_0(\beta\xi)$ in $\omega(\xi)$; (ii) $\log(\zeta)$ in ω_0; (iii) σ_m; (iv) the dependence of a upon F. Yao [2] assumes a pure stretching model, ignores the σ_0 contribution and derives

$$w_t = \frac{F}{2\pi pR}\left[\log\left(\frac{R+c}{c}\right)+\frac{R+2c}{R+c}\right] \tag{5}$$

Comparing with the present model, the followings are noted: (i) R and c are used here and both are linear, but a and c are squared in (4); (ii) σ_m is ignored and thus no cubic region of $F(w_t)$.

Conclusion

In summary, we derived rigorously a new model for AFM indentation on a microcapsule using classical linear elasticity that supercedes the existing theories. Our model is consistent with published data. Analyzing the typical data in literature, we have further shown that when the external load increased from null to a typical value, the elastic deformation switched gradually from pure bending to pure stretching. The new model is indispensable in mechanical characterization of a single thin-walled capsule.

References

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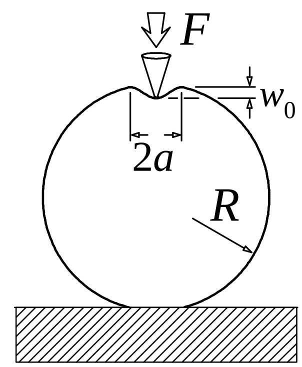

Figure 1. Schematic of an AFM indentation of a thin-walled microcapsule.

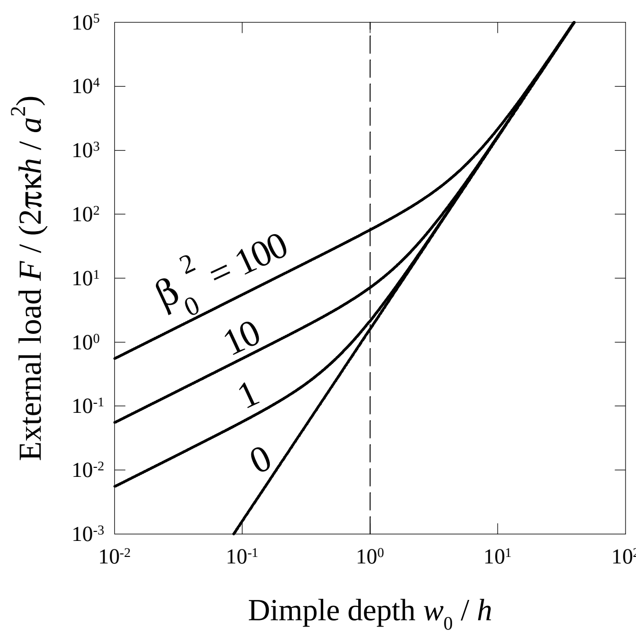

Figure 2. Mechanical response $F(w_0)$.