FLUID FLOWS THROUGH ANISOTROPIC, POROELASTIC BONE MODELS IN THE OPPOSITE DIRECTION TO THAT THROUGH ANALOGOUS ISOTROPIC MODELS

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INTRODUCTION

Biot's theory of poroelastic solids [1] describes the behavior of fluids in porous materials and provides the biophysical basis for load-induced fluid flow in bone. Applying this theory to bone, compression of the matrix causes an instantaneous increase in pore pressure within the matrix; to equilibrate this pore pressure gradient, fluid moves out of the pore spaces of the matrix under compression and back into the matrix upon subsequent load relaxation. Hence, bone essentially acts as a stiff, fluid-filled sponge.

We developed a three-dimensional (3D) transversely isotropic, poroelastic finite element (FE) model of the rat tibia to predict fluid movements induced by different mechanical loading regimes [2-4]. Both elastic properties and permeability were assumed to be transversely isotropic. Cortical bone stiffness is approximately 50 percent higher in the longitudinal direction than in the transverse direction [2]. Permeability of the fluid filled pericellular space, i.e. the lacunocanalicular system (LCS), is approximately one order of magnitude higher in the transverse direction than in the longitudinal direction [2]. In applying four-point bending loads to this transversely isotropic model tibia load-induced fluid movement occurs in a direction opposite to that described in the "bone sponge" analogy above; namely, interstitial fluid flow flows from the tensile toward the compressive aspect of the bone during load application, and vice versa during the load relaxation. Reverting to isotropic material parameters causes fluid to flow again as expected. After insuring that this unexpected effect was not due to a bug or modeling fluke, we embarked on a study to elucidate its cause and to understand its implications for convection-enhanced transport in bone.

METHODS

Two models of a poroelastic beam subjected to a pure bending load were developed to elucidate the cause of this effect. The geometry of these models was adapted from the models reported by Zhang and Cowin [6] and by Manfredini et al. [7], who previously analyzed an isotropic, poroelastic beam, representing a cortical bone specimen, subjected to a combined compression and bending load in an analytical and a FE study, respectively.

First, a 3D FE model was created in ABAQUS (Abaqus, Inc., Pawtucket, RI). The model consisted of 480 20-node, hexahedral pore pressure elements, similar to the model used in [7]. The main purpose of this model was to observe whether the same phenomenon occurs in beams of symmetric cross section. Then, a second model was programmed in Mathematica (Wolfram Research, Inc., Champaign, IL), based on the governing equations for a transversely isotropic poroelastic solid [8] and calculating the material coefficients according to the methods described in [9]. The partial differential equations were solved using the finite difference (FD) method.

Both models were subjected to cyclic loading with a loading frequency of 1 Hz. One load cycle was divided into 10 time steps and the pore pressure distribution was calculated for each time step. The material parameters were chosen within the range of the values published for cortical bone tissue of different species, e.g. Young's moduli for the longitudinal (E_z) and transverse directions (E_x, E_y): E_x, E_y=10 – 14 GPa, E_z=14-36 GPa, the Poisson's ratio ν_{xy}: 0.3-0.5, ν_{yz} and ν_{xz}: 0.16-0.32. The porosity ϕ of the beam was defined as a constant, 0.05, to reflect the porosity of the lacunocanalicular system. Pilot studies showed that the hydraulic permeability κ does not influence this counterintuitive flow effect directly but rather enhances the effect that is caused by other parameters. Thus, permeability was assigned a constant value of 1.5×10^{-20} m for the purpose of these calculations.

A key parameter in this study was the solid bulk modulus K_s. Based on the assumption that the anisotropy of cortical bone can be explained by the arrangement of the porous spaces within bone [10], and that the bone matrix per se is isotropic, we calculated the solid bulk modulus as a function of the apparent elastic moduli E_x and the lacunocanalicular porosity ϕ of cortical bone.

RESULTS

The pore pressure distribution in a symmetric, poroelastic, anisotropic beam was calculated using the FE model (Fig. 1). This result confirmed that the counterintuitive fluid flow direction is not an effect of the asymmetric geometry of the rat tibia.

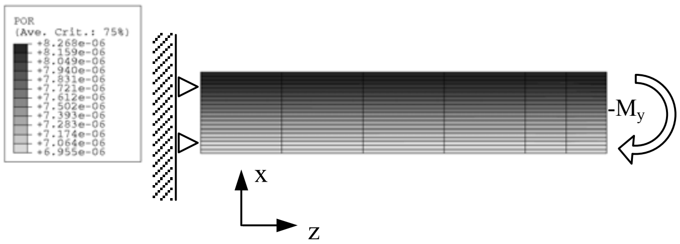

Figure 1: Pore pressure distribution in a poroelastic beam subjected to a pure bending load. A darker grey value corresponds to a higher value, which means that the fluid flows from the darker towards the brighter areas.

Using the FD model, we then determined the flow direction for different material parameter configurations. Keeping the Poisson's ratios constant, the fluid flow direction changed if the ratio between the Young's moduli reached a certain threshold. This threshold was determined for each configuration and was depicted graphically. A collection of the threshold curves for a number of configurations is shown in Fig. 2.

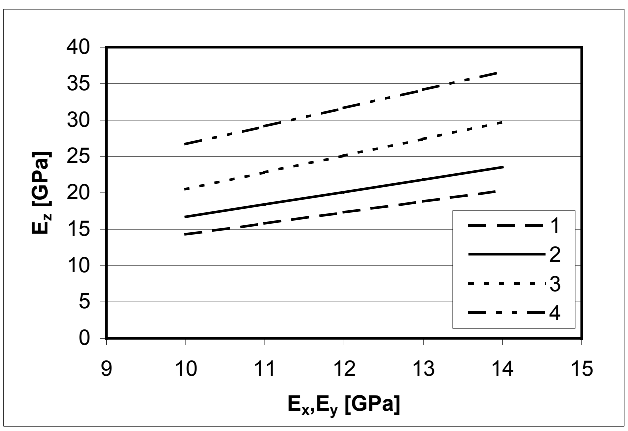

Figure 2: The threshold curve for the following configurations: Conf. 1: (ν_{xy}=0.58, ν_{yz}= ν_{xz}=0.31), 2: (0.49, 0.3), 3: (0.3, 0.3), 4: (0.5, 0.2). If the ratio $E_z/E_x,E_y$ is above the curve for a given configuration, a counterintuitive fluid flow direction is predicted.

DISCUSSION AND CONCLUSIONS

Fluid flows from the tensile cortex to the compressive cortex during compression of a poroelastic, anisotropic beam subjected to a pure bending load, regardless of beam geometry. Based on the results of the FD model, it was shown that the ratio of the Young's modulus in the longitudinal direction to that in the transverse direction is critical in determining direction of flow; above a certain threshold, the flow direction changes from the intuitive compression $\rightarrow$ tension direction to the opposite direction. It is as if the beam undergoes "internal buckling", whereby the volume expansion in the transversal plane exceeds the compression in the longitudinal direction. Consequently, the threshold value (ratio) can be further reduced by "weighting" the transversal Poisson's ratio toward incompressibility ($\nu_{xy}\rightarrow 0.5$). It is important to note that these threshold ratios are not hypothetical and unrealistic values, but are reached with common elastic parameters for cortical bone, as reported in the literature.

The solid bulk modulus, K_s plays an important role as well. To our knowledge, there is no reported *measured* value for K_s of bone in the literature. Therefore, it has to be calculated from other known elastic parameters. Previously, when bone was treated as an isotropic poroelastic material, K_s is typically calculated as a macroscopic parameter of bone tissue (e.g. [5]). In this study (based on [9] and [10]), we calculated K_s as microscopic parameter of the solid bone phase.

This is the first published report to our knowledge that describes the phenomenon of interstitial fluid movement in the opposite direction through anisotropic, poroelastic solid beams. Previously, the anisotropy of bone tissue has been considered to be of minor significance for load-induced fluid flow in bone [5]. Given the results of this study, not only does the inherent anisotropy of bone tissue have profound implications for fluid flow direction through bone (which in turn modulates chemotransport through bone), but it also provides impetus to reexamine isotropic poroelastic modeling studies of the past.

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