COMPARISON OF FINITE VOLUME, FINITE ELEMENT AND THEORETICAL PREDICTIONS OF BLOOD FLOW THROUGH AN IDEALISED FEMORAL ARTERY

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INTRODUCTION

Advances in Computational Fluid Dynamics (CFD) and computer technology are increasingly allowing the simulation of blood flow in the cardiovascular system. CFD software provides a fast, non–invasive method of assessing surgical techniques [1,2], analysing arterial diseases [3] and examining blood flow in a number of diseased conditions [4].

Current software is based on two principle numerical methods – Finite Element Method (FEM code) and Finite Volume Method (FVM code). Both methods involve subdividing the flow domain into a large number of finite elements / control volumes and then solving the governing equations of fluid flow i.e. the 3-D Navier-Stokes equations [5]. In the process a system of algebraic equations is formed and subsequently solved by an iterative method. The numerical methods differ in their derivation and definition of these algebraic equations.

FEM use simple piecewise functions (e.g. linear or quadratic) to describe the local variations of unknown flow variables, φ. The Navier-Stokes equations are precisely satisfied by the exact solution, φ. If the piecewise approximating functions for φ are substituted into the equation it will not hold exactly and a residual is defined to measure the errors. The residuals are next minimized in some sense by multiplying them by a set of weighing functions and integrating. As a result a set of algebraic equations for the unknown coefficients of the approximating functions is obtained [6].

For FVM, a formal integration of Navier-Stokes equations over all the control volumes of the solution domain is carried out. A variety of finite-difference-type approximations for the terms in the integrated equation representing flow processes such as convection, diffusion and sources are then applied. This converts the integral equations into a system of algebraic equations [7].

This study is concerned with comparing two software packages Adina 7.5(FEM) and Fluent Europe 6.0(FVM) to exact theoretical solutions for two models. The models consist of a femoral artery modelled as a rigid straight tube with steady/ pulsatile flow. The resulting velocity profiles are compared to Poiseuille/ Womersley flow theory respectively [8].

METHODS

Geometric Model

The femoral artery is modelled as a rigid straight cylindrical tube of constant diameter 0.006m and length 0.1m. Blood is modelled as an incompressible homogeneous Newtonian fluid with a viscosity of 0.00345 Pas and density of 1050 kg/m^3.

Flow Conditions

For both models ideal Dirichlet boundary conditions were applied at the inlet. The steady flow inlet condition consisted of a velocity of 0.1m/s. This represents a time average of the femoral pulse. The pulsatile velocity input consisted of a simplification of the femoral artery waveform [9] i.e. a sine wave of amplitude 0.1m/s and period 0.756s superimposed on a steady wave of 0.1m/s. For both models no slip boundary conditions were applied at the artery wall and Neumann boundary conditions were applied at the outlet.

Model Independence

A prerequisite for all CFD studies is that the model under investigation be grid, period and time step independent i.e. independent of the size of the control volumes/elements, number of cycles of the input velocity and the time step between which the software predicts a solution. Grid independence was achieved by running successively smaller volumes/elements until a percentage difference of less than 2% was observed between the resulting velocity profiles. Period independence was achieved by applying an input velocity of ten cycles. Results from successive cycles at various phases were compared until a percent difference of less than 2% was achieved. Running models with successively smaller time steps until the results showed a percentage difference of less than 2% attained time step independence.

RESULTS

In this study results were taken at a location in the artery 0.075m downstream from the inlet. This was to ensure that the results represented fully developed flow for both steady and unsteady models. The results show that FVM and FEM methods are qualitatively similar but quantitatively different from the theoretical solutions, with the

FVM providing a better approximation to theory (see figures 1 & 2). This observation holds for both steady and unsteady flow. Grid independence was achieved at 90,000 elements (FEM) and 140,000 volumes (FVM). Consequently the FVM solution required approximately double the computational storage memory and double the run time e.g. steady flow 45min(FVM), 20min(FEM). For unsteady flow period independence was obtained after 2 cycles and with a time step of 0.00756s

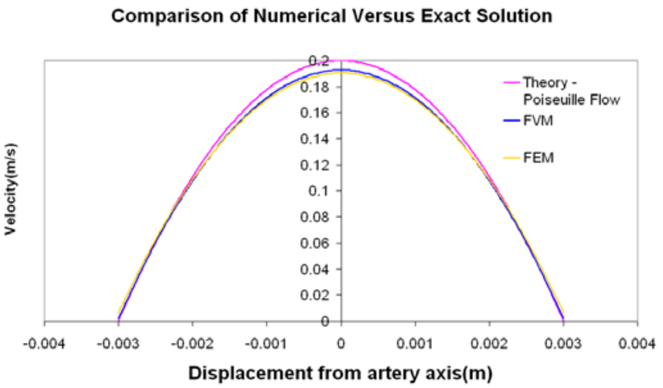

Figure 1: Velocity profiles for steady flow

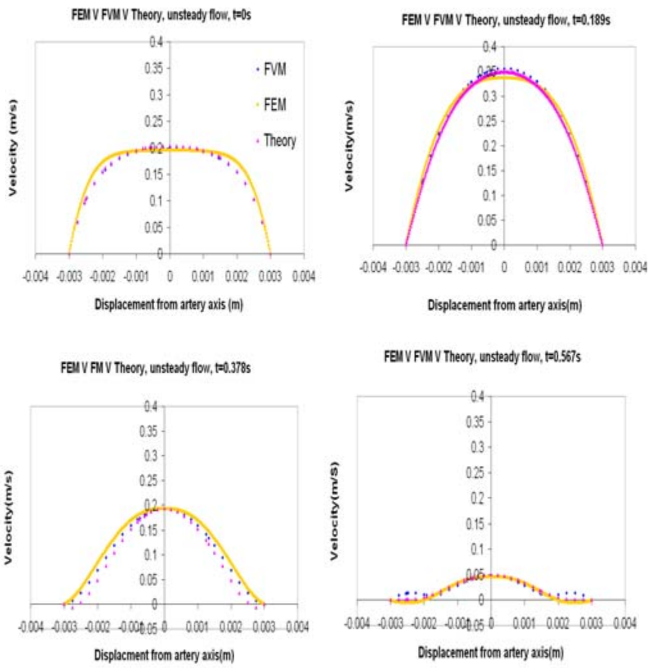

Figure 2: Velocity profiles 0.075m downstream of the inlet at four phases of the input velocity waveform a) Starting time b) Maximum positive velocity c) Saddle point and d) Maximum negative velocity

DISCUSSION

The results demonstrate the accuracy of two commercially available software packages compared to exact analytical solution for blood flow in an idealized femoral artery. Differences arise due to round off errors in the software and approximations used in the models. Fluent Europe 6.0 is deemed to be slightly more accurate. The choice of software also depends on computer and time considerations. Adina 7.5 was observed to be faster and required less computational memory.

Given the level of CFD studies that have been carried out over the past decade and the implications of their results, the validation of the code needs to be continually assessed and extended to more complicated geometries e.g. end-to-side junction. It is concluded that current numerical methods are approaching exact solutions, but still require some refinement.

ACKNOWLEDGES

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