THE EFFECTS OF POROSITY AND FIXED CHARGE DENSITY ON HYDRAULIC PERMEABILITY OF CARTILAGE

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INTRODUCTION

Interstitial fluid flow plays very important roles in the viscoelastic behavior of cartilage, and in the transport of nutrient inside tissue [1]. The interstitial fluid flow in cartilage is governed by the hydraulic permeability. Two methods are commonly used for determining hydraulic permeability of the cartilage samples: one is a direct permeation test and the other is an indirect measurement using curve-fitting of creep or stress relaxation data to the biphasic model [2]. However, since cartilage is a negatively charged tissue, the mechno-electrochemical coupling will affect the hydraulic permeability obtained by biphasic curve-fitting. Thus, the main objective of this study is to numerically investigate the effect of fixed charge density (FCD) on the apparent (biphasic) hydraulic permeability based on the tripahsic mechano-electrochemical theory [3].

METHODS

One-dimensional confined creep test, as shown in Figure 1, was simulated by finite element method (FEM) based on the triphasic theory [3,4]. The resultant triphasic creep curve was fitted to the biphasic model to obtain the apparent biphasic permeability and aggregate modulus. The FCD was varied in the numerical simulation in order to investigate the effect of FCD on apparent biphasic permeability.

Governing equations

The one-dimensional triphasic governing equations for the confined creep test are given by [3,4]:

$$\partial\sigma/\partial z = 0 \tag{1}$$

$$\partial v^s/\partial z + \partial J^w/\partial z = 0 \tag{2}$$

$$\partial\left(\phi^w c^+\right)/\partial t + \partial J^+/\partial z + \partial(\phi^w c^+ v^s)/\partial z = 0 \tag{3}$$

$$\partial\left(\phi^w c^-\right)/\partial t + \partial J^-/\partial z + \partial(\phi^w c^- v^s)/\partial z = 0 \tag{4}$$

where σ is the total stress of the mixture, J^w is the water flux, J^+ and J^- are the fluxes of cation and anion, respectively. These quantities are related to solid displacement u, modified chemical potentials of water ε^w, cation ε^+ and anion ε^-, and given by [5]:

$$\sigma = -\left[RT\varepsilon^w + RT(c^+ + c^-)\right] + (\lambda + 2\mu)\frac{\partial u}{\partial z} \tag{5}$$

$$J^w = -RTk\left(\frac{\partial\varepsilon^w}{\partial z} + \frac{c^+}{\varepsilon^+}\frac{\partial\varepsilon^+}{\partial z} + \frac{c^-}{\varepsilon^-}\frac{\partial\varepsilon^-}{\partial z}\right) \tag{6}$$

$$J^+ = c^+ J^w - \frac{\phi^w c^+ D^+}{\varepsilon^+}\frac{\partial\varepsilon^+}{\partial z} \tag{7}$$

$$J^- = c^- J^w - \frac{\phi^w c^- D^-}{\varepsilon^-}\frac{\partial\varepsilon^-}{\partial z} \tag{8}$$

where λ and μ are Lame coefficients of the solid matrix, solid velocity $v^s = \partial u/\partial t$, strain $e = \partial u/\partial z$, porosity $\phi^w = (\phi_0^w + e)/(1+e)$, fixed charge density $c^f = c_0^f/(1 + e/\phi_0^w)$, cation and anion concentrations $c^+ = 0.5*\left(c^f + \sqrt{\left(c^f\right)^2 + 4\varepsilon^+\varepsilon^-}\right)$, $c^- = 0.5*\left(-c^f + \sqrt{\left(c^f\right)^2 + 4\varepsilon^+\varepsilon^-}\right)$.

Note that ϕ_0^w and c_0^f are the porosity and FCD at initial state, D^+ and D^- are the diffusivities of cation and anion, respectively.

Initial condition:

t=0: $u = 0;\ \varepsilon^w = \varepsilon^{w*};\ \varepsilon^+ = \varepsilon^{+*};\ \varepsilon^- = \varepsilon^{-*}$

where * stands for the values in the bathing solution.

Boundary condition:

z=0: $u = 0;\ J^w = J^+ = J^- = 0$

z=h: $\sigma = -\sigma_0$; $\varepsilon^w = \varepsilon^{w^*}$; $\varepsilon^+ = \varepsilon^{+^*}$; $\varepsilon^- = \varepsilon^{-^*}$

where σ_0 is the applied constant stress.

Biphasic curve fit

The resultant triphasic creep curve was fitted by the following biphasic equation to obtain the apparent biphasic permeability k_{app} and aggregate modulus H_A [2],

$$\frac{u(t)}{h} = \frac{\sigma_o}{H_A}\left[1 - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{2})^2}\exp\left(-\frac{H_A k_{app}(n+\frac{1}{2})^2\pi^2}{h^2}t\right)\right]. \qquad (9)$$

RESULTS AND DISCUSSION

The following physiologically realistic parameters for cartilage were used in our numerical simulations: $c_0^* = 0.15\text{MPa}$, $\lambda + 2\mu = 0.4\text{MPa}$, $D^+ = 0.5\times10^{-9}\ \text{m}^2/\text{s}$, $D^- = 0.8\times10^{-9}\ \text{m}^2/\text{s}$, $\phi_0^w = 0.82, 0.87$, $c_0^f = 0 \sim 0.45\text{mEq/ml}$, $\sigma_0 = 0.028\text{MPa}$, $h = 1\text{mm}$. The triphasic permeability was determined by the following equation: $\kappa = a(\phi^w/\phi^s)^n$, where a =0.00339 nm^2, n =3.236 and κ is Darcy permeability [6].

Figure 2 shows the creep behaviors of cartilage as a function of FCD. The tissue becomes stiffer with increasing FCD. Curve-ftting of these results to equation (9) resulted in the apparent permeability as a function of FCD, see Figure 3. The apparent biphasic permeability in Figure 3 was normalized by the permeability value at c^f = 0.1mEq/ml (ϕ_o^w = 0.87). With increasing FCD, the tissue appears less permeable. For lower water content, the effect of FCD on apparent permeability is not as significant as for higher water content. The experimental results for cartilage permeability [7] were also plotted in Figure 3 for comparison. Note that in these experimental results, both effects of water content and FCD were included [7]. From Figure 3, one could see that a small variation in porosity could cause significant change in permeability. Thus, the water content plays a more important role in permeability than the FCD for cartilage tissues.

ACKNOLOWLEDGEMENT

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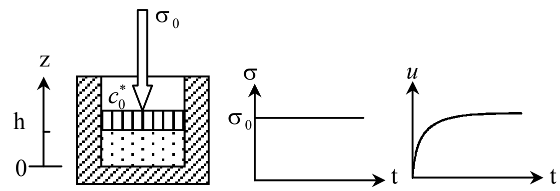

Figure 1. Schematic of a confined creep test.

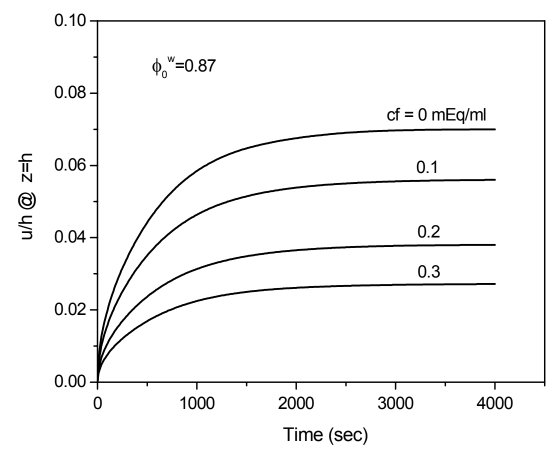

Figure 2. Triphasic creep curve at different FCDs

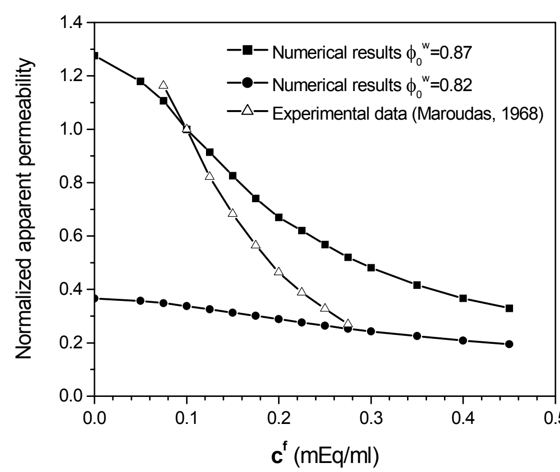

Figure 3. Effects of porosity and FCD on permeability