CONJUGATE MOTION ANALYSIS AND ITS APPLICATION IN HUMAN AND ANIMAL'S MULTI-POINT CONJUGATION JOINTS

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INTRODUCTION

Conjugate motion is a kind of motion, which is geometrically constrained through contacts (conjugation) between boundary surfaces. These boundary surfaces are called conjugate surfaces that should keep contact with each other during the whole motion. The conjugate motion obeys the *Fundamental Equation of Conjugation*, and involves two or more bodies. The motions encountered in biomechanical system are generally not free body motions, but are conjugate motions. Meanwhile, the joints encountered in human/animal skeleton, are not lower-pair joints of ordinary mechanisms, but are multi-point conjugation (*MPC*) joints. Hence, there is a need to investigate the conjugate motions of multi-point conjugation joints. However, it has not yet been generally recognized.

THEORY OF CONJUGATE SURFACES

The ***Theory of Conjugate Surfaces*** [1], discusses the laws of inter-relationship between two entities. One entity is a pair of geometrical configurations, the base and the mate surface, which form the boundaries of two contacting solids, for example, femur and tibia in the knee joint, respectively. Another entity is the conjugate motion, which is defined as a such relative motion between the two relevant solids, that the continuous contact between the boundaries of the two solids is guaranteed. The theory consists of 4 parts. They are *Solid body motion representation: rotation of vector*, *Kinematical representation of geometry: engineering differential geometry*, *Kinematics of conjugate motion*, and *Geometry of conjugate configurations*. With the aid of the theory, any (mate) surface can be expressed as the envelope surface generated by a prescribed base surface together with a relevant conjugate motion. Meanwhile, the theory also provides an efficient algorithm for computing the relevant conjugate motion when the relevant conjugate surfaces are known.

UNIQUE FEATURES

Two basic features of instantaneous conjugate motion [2]***:*** At any instant of a conjugate motion, there exists at least one instantaneous conjugation point (*ICP*) between a pair of conjugate surfaces. Instantaneous conjugate motion of any instant can be specified by a bi-vector, which is the aggregation of the translational velocity and the angular velocity of the relevant instant. For keeping continuous conjugation between the two conjugate surfaces at the relevant instantaneous conjugation point P, the relative sliding velocity $\boldsymbol{\lambda}_p$ at P must be perpendicular to their common normal $\boldsymbol{N}_p$.

$$\begin{aligned}\boldsymbol{\lambda}_p \cdot \boldsymbol{N}_p = & A_p[\, v\cos\varepsilon\cos\delta + \omega\,(z_p\cos\phi\sin\theta - y_p\sin\phi\,)] \\ & + B_p[\, v\cos\varepsilon\sin\delta + \omega\,(x_p\sin\phi - z_p\cos\phi\cos\theta\,)] \\ & + C_p[\, v\sin\varepsilon + \omega\,(y_p\cos\phi\cos\theta - x_p\cos\phi\sin\theta\,)] = 0 \qquad (1)\end{aligned}$$

Equation (1) is called the ***Fundamental Equation of Conjugation***, and it is a characteristic of conjugate motion. Its alternative form is as Eq. (2).

$$\begin{aligned}(\, v/\omega\,) = & [\, A_p(y_p\sin\phi - z_p\cos\phi\sin\theta\,) \\ & + B_p(z_p\cos\phi\cos\theta - x_p\sin\phi) \\ & + C_p(x_p\cos\phi\sin\theta - y_p\cos\phi\cos\theta\,)] \\ & /\, [\, A_p\cos\varepsilon\cos\delta + B_p\cos\varepsilon\sin\delta + C_p\sin\varepsilon\,] \qquad (2)\end{aligned}$$

Every conjugation point contributes one constraint equation between 6 instantaneous motion parameters (*imps*). Two basic features can be deduced from Eq. (2). The first is that the 6 *imp*s are partitioned into two groups. One group consists of 2 magnitude-*imp*s, v, ω, and appears only on the left side of the equation. The other group consists of 4 orient-*imp*s, ε, δ, ϕ, θ, and appears only on the right side. The second feature is that the 2 magnitude-*imp*s are linearly proportional to each other with their proportional constant being a function of the 4 orient-*imp*s. These two basic features are unique for the instantaneous conjugate motion.

MULTI-POINT CONJUGATION JOINT

The joints in bio-mechanism, in anatomy, are all multi-point conjugation joints. Figure 1(a) shows an inter-vertebral joint in the poultry neck. It is a four-point-conjugation joint between the two neighboring bones 1 and 2, and there exist 4 simultaneous *ICP*s at any instant. The conjugate motion between the two bones is constrained by four parallelly disposed, point-conjugation pairs. And its number of degrees of freedom (*d.o.f.*) can be calculated to be *2* [3,4]. Figure 1(b) shows the skeleton of snake vertebrae. Figure 2 shows the skeleton of pig's foreleg in pre-assembled and assembled state. In the latter state, a multi-point conjugation joint is obtained.

(a) (b)

Figure 1. (a) Bones of Poultry's Cervical Vertebrae (b) Bones of Snake Vertebrae

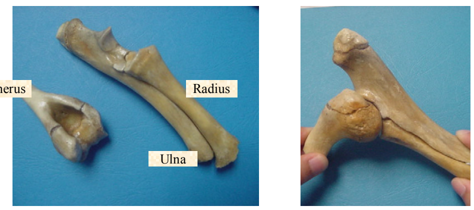

Figure 2. Bones of Pig's Foreleg

EXEMPLARY BIOMECHANICAL SYSTEM AND DISCUSSION

Figure 3 shows a bio-mechanical system of simple 2-branch connection. The geometrical sketch is shown in the center. The instantaneous conjugate motion of body II, as expressed by a bi-vector is shown at right side, while the instantaneous motion of body M is shown at the left side. The chain graphs for the 2-branch connection are: *I – M – II and I – II*. Body *I* is the base (reference) body and body *II* the moving body. Body *M* is the medium body. *O* is the base (reference) point. Then, the following set of equations of global compatibility, expressed in terms of bi-vectors, can be derived.

$$v\boldsymbol{v} = v_{11}\,\boldsymbol{v}_{11} = v_{12}\,\boldsymbol{v}_{12} + v_{22}\,\boldsymbol{v}_{22}$$
$$\omega\boldsymbol{\omega} = \omega_{11}\,\boldsymbol{\omega}_{11} = \omega_{12}\,\boldsymbol{\omega}_{12} + \omega_{22}\,\boldsymbol{\omega}_{22} \qquad (3)$$

In this bio-mechanical system, the total number of mediums is 1 and the total number of joints is *3*. The 3 joints are two *4*-point conjugation joints and one *2*-point conjugation joint. Since there are *4* constraint equations for a *4*-point joint and 2 constraint equations for a *2*-point joint, the following Eq. (4) can be derived from these constraint equations.

$$v_{11} = f_{11v}(\theta_{11})\omega_{11}\,,\quad \boldsymbol{\omega}_{11} = \boldsymbol{u}_{11\omega}(\theta_{11}),\quad \boldsymbol{v}_{11} = \boldsymbol{u}_{11v}(\theta_{11})$$
$$v_{12} = f_{12v}(\theta_{12})\omega_{12}\,,\quad \boldsymbol{\omega}_{12} = \boldsymbol{u}_{12\omega}(\theta_{12})\,,\quad \boldsymbol{v}_{12} = \boldsymbol{u}_{12v}(\theta_{12})$$
$$v_{22} = f_{22v}(\phi_{22},\theta_{22},\delta_{22})\omega_{22}\,,$$
$$\boldsymbol{\omega}_{22} = \boldsymbol{u}_{22\omega}(\phi_{22},\theta_{22},\delta_{22})\,,\quad \boldsymbol{v}_{22} = \boldsymbol{u}_{22v}(\phi_{22},\theta_{22},\delta_{22}) \qquad (4)$$

Finally, the instantaneous conjugate motion ($v\boldsymbol{v}$, $\omega\boldsymbol{\omega}$) of body *II* relative to body *I* can be solved from Eqs. (3) and (4) by using the two basic features of instantaneous conjugate motion, as follows.

$$v\boldsymbol{v} = f_{11v}(\theta_{11})\,\omega_{11}\ \boldsymbol{u}_{11v}(\theta_{11})$$
$$\omega\boldsymbol{\omega} = \boldsymbol{u}_{11\omega}(\theta_{11})\,\omega_{11} \qquad (5)$$

Equation (5) shows that the instantaneous conjugate motion depends on two parametric variables ω_{11} and θ_{11}, and the motion is of 2 d.o.f.

in terms of the relevant independent *imps*. A computer program has been developed for design and motion analysis of this very useful bio-like (arm-like) mechanism. This computer program may be beneficial to the paleontologists for inferring Paleozoic animal's posture and movement [5], and may be also helpful for explaining why the patient after arthrodesis and knee joint replacement surgery experiences less tolerance to mobility [6].

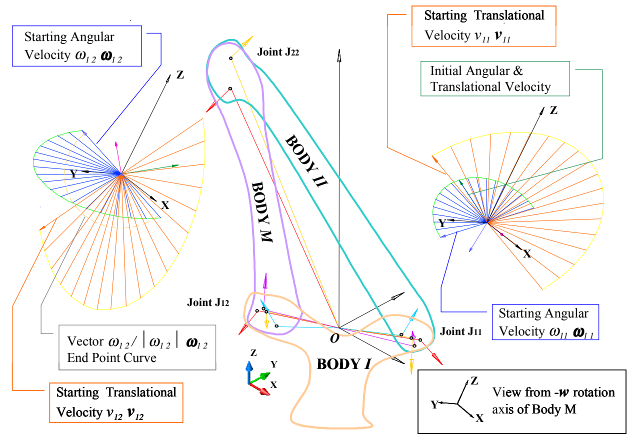

Figure 3. Motion Envelops of Arm-like Mechanical System

CONCLUSIONS

Multi-point conjugation joints are ubiquitously encountered in nature. These joints are more flexible in motion control, and more tolerable / adaptable to geometrical deviations, but are inadequately investigated. In this investigation, the methodology for instantaneous conjugate motion analysis is elaborated with reference to the theory of conjugate surfaces and the two basic features of instantaneous conjugate motion. With the aid of this methodology, the instantaneous conjugate motion between any two bodies in a bio-mechanical system can be readily analyzed, as illustrated in this paper by the motion analysis of an exemplary bio-mechanical system.

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