DETERMINATION OF POISSON'S RATIO OF ARTICULAR CARTILAGE IN INDENTATION TEST USING DIFFERENT SIZED INDENTERS

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INTRODUCTION

Indentation has been commonly used in testing the mechanical properties of articular cartilage, and its loading condition is considered more relevant to the physiological condition. Unlike in simple tension or compression tests, in indentation test, the material constants are coupled in the indentation load-deflection relations, and additional independent tests are needed to uncouple them. In previous studies, confined compression and torsion test[1], as well as directly measuring the lateral expansion in unconfined compression[2] have been used along with indentation test. There are two issues that may impede the application of such procedures. Firstly, the discrepancy between material constants obtained from different test geometry has been observed. As a result, a material constant measured or calculated in a certain test geometry may not be applicable to others[2]. Secondly, some samples, such as mouse patellar cartilage, could be too small to be handled and tested by means other than indentation. A method of determining all the material constants by a single indentation test had been achieved in the context of biphasic theory[3]. The method has been applied in many works to determine all the material parameters required by biphasic models. In general, the values of Poisson's ratio reported in these works have been small, the mean value typically ranging from 0.0 to 0.28[3,4]. The small Poisson's ratio values have been confirmed by the direct measurements in unconfined compression[2]. However, it has also been shown that applying the Poisson's ratio directly measured in unconfined compression to the indentation will result in a higher Young's modulus in comparison with that from the confined and unconfined compression[2]. In this study, we introduce a method for determining Poisson's ratio and Young's modulus of soft materials in indentation using different sized indenters, without requiring assistance from other test methods. The method was firstly validated on polyurethane rubber and a elastic foam with Young's moduli known from unconfined compression. Then, it was applied to bovine cartilage, to obtain the Young's modulus and Poisson's ratio in instantaneous and equilibrium response, where the cartilage can be treated as a single-phase elastic media. A basic finding was that the Poisson's ratio in indentation geometry obtained in this work was consistently higher than those directly measured in unconfined compression and those derived from biphasic indentation theory. Using finite element simulations, we were able to attribute this discovery to the inhomogeneity and anisotropy of the cartilage.

METHODS AND MATERIALS

The method is derived from the solution developed by Hayes et al[5], for a flat-ended cylindrical rigid punch with radius a indenting on the surface of a linear elastic layer with thickness h, shear modulus G and Poisson's ratio ν. The layer is bonded onto a flat rigid substrate. The indentation stiffness that is defined as the ratio of the indenting load p to the indenting depth ω, in ideal linear elastic cases may be expressed as:

$$\frac{p}{\omega} = \frac{4Ga}{1-\nu}\,\kappa(a/h,\nu) \tag{1}$$

where, $\kappa(a/h,\nu)$ is a correction factor that accounts for the finite layer effect. Under the same condition, and following the same procedure, indenting the sample twice using two different sized indenters with radius a1 and a2 respectively, we have:

$$\left(\frac{p}{\omega}\right)_1 \Big/ \left(\frac{p}{\omega}\right)_2 = \frac{a_1}{a_2}\frac{\kappa(a_1/h,\nu)}{\kappa(a_2/h,\nu)} \tag{2}$$

In Eq. (2), Poisson's ratio is the only unknown and it can be obtained by solving this nonlinear equation. Once the Poisson's ratio is obtained, the shear modulus can be recovered using Eq. (1). Indentation tests on cartilage samples were performed by indenting with flat-ended cylindrical indenters to 0.15 mm at a nominal loading rate 1.5mm/sec, and holding for 1200 seconds. The actual loading rate for each test was checked to assure the uniformity of loading rate. Testing was done on an EnduraTec 3200 system. The stiffness during loading that obtained by linearly fitting the loading part of the load-depth curve was designated the instantaneous stiffness, the stiffness at 1200 second the equilibrium stiffness, they are related by:

$$\left(\frac{p}{\omega}\right)_{\infty} = \left(\frac{p}{\omega}\right)_{o} H(t=\infty) \qquad (3)$$

the subscripts o and ∞ represent the instantaneous and equilibrium entity, respectively. $H(t)$ is the load relaxation function obtained by normalizing the load relaxation history with its peak The instantaneous and equilibrium Poisson's ratio can then be calculated using Eq.(2) and the indentation stiffness. Verification tests were performed on a polyurethane rubber and an elastic foam. The Young's moduli of the two materials determined in indentation and unconfined compression in small deformation range (5% compressive strain for rubber, 3%for foam) are shown in Table 1. The relative error between the Young's moduli determined from the two methods was 6.5% for the rubber and 3.3% for the foam. The Poisson's ratios for the two materials are also shown in Table 1. For cartilage tests, 7 square shaped samples (20X20mm) were cut from the relatively flat part of 7 cow patellae. Each sample was indented three times using flat-ended cylindrical indenters with radius 2mm, 4mm and 6mm, respectively. Cartilage thickness was measured with a sharp tipped thickness detector, as described in reference[3]. The load and displacement history data were collected from each test.

Material	Indenter Size (mm)	Poisson's ratio (Indent)	Young's Modulus (Mpa)	
			Indent	Compress
Elastic Foam	13.0/6.0	0.347	0.432	0.418
Urethane Rubber	6.0 / 4.0	0.489	4.420	4.712
	6.0 / 2.0	0.489	4.416	
	4.0 / 2.0	0.490	4.402	
	Average	0.489	4.413	

Table 1. Elastic Properties of Urethane Rubber and A Elastic Foam Obtained From Indentation and Unconfined Compression Tests

In order to understand the effects of inhomogeneity and anisotropy, a finite element model was created to simulate the process of an elastic layer indented by different sized indenters. The anisotropy and inhomogeneity of the elastic layer was modeled respectively. In the transversely isotropic case, the equivalent Poisson's ratio depends on the ratio of Young's moduli, ratio of shear moduli and two Poisson's ratios, namely, E_p/E_t, G_p/G_t, ν_p and ν_{tp} [8], here p and t represent in-plane and out-plane entity, respectively. In inhomogeneous case, the elastic layer consisted of three homogeneous and isotropic sublayers with the thickness of 10%, 60% and 30% of the layer thickness, respectively, simulating the superficial, the middle and the deep layers of cartilage. Using the indentation stiffness data generated by the FE model, we were able to find the equivalent homogeneous and isotropic Poisson's ratio, as well as the elastic moduli.

RESULTS

Table 2 shows the predicted Poisson's ratio Young's modulus from one cartilage sample. The values of Young's modulus found in our tests were 1.79 ± 0.59 Mpa in instantaneous response, and 0.45 ± 0.26 Mpa in equilibrium. The values of Poisson's ratio were 0.503 ± 0.028 in instantaneous response and 0.463 ± 0.073 in equilibrium. In the anisotropic FE model, we found that the equivalent Poisson's ratio increased with the increase of E_p/E_t, ν_p and ν_{tp}, and changed only slightly with the variation of G_p/G_t. When the material exhibits strong anisotropy (E_p/E_t >5.0, for example), the equivalent Poisson's ratio could be close to 0.50 even if ν_p and ν_{tp} are as small as 0.10. Using the experimental data provided in reference[6] and assigning 0.49 to the instantaneous Poisson's ratio for each sublayer, we were able to show that the equivalent isotropic and homogeneous Poisson's ratio could exceed 0.50 in instantaneous response; and in equilibrium state, it was greater than 0.40, significantly higher than that of individual sublayers (0.08, 0.32 and 0.16 for superficial, middle and deep layers in equilibrium), it was also significantly higher than that obtained from unconfined compression FE simulation (0.23). The dependency of predictions on the indenter sizes was also observed in both inhomogeneous and anisotropic finite element simulations.

Indenter Size (mm)	Poisson's Ratio ν		Young's Modulus E (Mpa)	
	t=0	t=∞	t=0	t=∞
6.0 & 4.0	0.482	0.333	1.913	0.512
6.0 & 2.0	0.466	0.326	2.111	0.519
4.0 & 2.0	0.447	0.320	2.207	0.523
Average	0.465	0.326	2.077	0.518

Table 2 A Bovine Cartilage Indentation Test Result, t=0 for instantaneous response, and t=∞ the equilibrium

SUMMARY AND DISCUSSION

A method for determining the Poisson's ratio and Young's modulus of articular cartilage and other soft materials from indentation using different sized indenters has been introduced. The effectiveness of this method has been shown in rubber and elastic foam tests. Applying the method to bovine cartilage, we were able to determine the equivalent elastic properties of the tissue in both instantaneous and equilibrium states. The equilibrium Young's modulus found in our tests was consistent with those in [7]. The Poisson's ratio found in our tests may be compared with some previous works[5], however, it was significantly higher than those directly measured in unconfined compression. Out-of-range Poisson's ratio values (>0.50) and indenter size dependency of the predictions were also observed for some samples. Using FE simulation, we were able to show that, for an anisotropic and inhomogeneous elastic layer, its equivalent Poisson's ratio and Young's modulus in indentation geometry are not comparable to any individual anisotropic Poisson's ratio and Young's modulus. We also discovered that the anisotropy and inhomogeneity could be responsible for the high values of Poisson's ratio found in this work. Those findings suggest that, for the cartilage exhibiting relatively weak inhomogeneity and anisotropy, it could be properly modeled as homogeneous, isotropic and linear elastic material in instantaneous and equilibrium states, and the introduced method may be used to determine the associated Poisson's ratio and elastic moduli; however, in order to adequately model those with strong inhomogeneity and strong anisotropy, more sophisticated material and structural models have to be employed.

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