FINITE REYNOLDS NUMBER EFFECTS ON STEADY PROPAGATION OF A LIQUID PLUG IN A 2D-CHANNEL

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INTRODUCTION

Liquid is instilled into the pulmonary airways in some medical treatments such as surfactant replacement therapy (SRT), partial liquid ventilation (PLV), and drug delivery. The formation of a liquid plug in the trachea, before inspiration, is important in creating a more uniform liquid distribution throughout the lung [1]. As the plug propagates with air during the inspiration, it deposits a trailing liquid film on the airway wall and may eventually rupture. The higher homogeneity of the installed liquid distribution in the lung will be achieved by delivering the plug into lower generation of the airway without rupture.

Howell et al. [2] asymptotically analyzed a liquid plug propagation in a flexible tube. They identified a critical imposed pressure drop above which the liquid plug will eventually rupture. Waters and Grotberg [3] asymptotically analyzed surfactant laden liquid plug propagation. They showed that the driving pressure, ΔP for a given Ca increases with increasing the surface elasticity, but decreases with the precursor film thickness. The trailing film thickness increases with ΔP, but at a slower rate when the surface elasticity is larger. Heil [4] numerically investigated inertia effects in a semi-infinite bubble propagation. He showed a non-monotonic variation of the trailing film thickness with changes in Re at fixed Ca.

In this study we investigate the effect of the fluid inertia in the steady propagation of a liquid plug within a two-dimensional channel lined by a uniform, thin liquid film. Also we examine the effect of the plug length on the system at finite Reynolds number.

MODEL

We investigated the steady propagation of a liquid plug inside a 2-dimensional channel of width 2H as shown in Fig.1, numerically. A pressure difference between the front and back air finger, $\Delta P = P_1 - P_2$, drives the liquid plug of a homogeneous, incompressible, Newtonian fluid with constant speed U. The plug length, L_p is defined as the distance between the two meniscus tips. We assumed the problem is symmetrical about the channel centerline and only the lower half of the domain is considered. The precursor film thickness, h_2, is defined to be equal to the trailing film thickness, h_1, because of the steady state. In a moving frame with constant velocity U of both meniscus tips, the flow inside the liquid plug is described by the steady, non-dimensional Navier-Stokes and continuity equations,

$$\mathrm{Re}(\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nabla^2\mathbf{u}, \qquad \nabla\cdot\mathbf{u} = 0 \tag{1}$$

where $\mathbf{x}=\mathbf{x}^*/H$, $\mathbf{u}=\mathbf{u}^*/U$ and $p=p^*/(\mu U/H)$, $\mathrm{Re}=\rho UH/\mu$, ρ is the liquid density and μ is the viscosity. The kinematic boundary condition on the air-liquid interfaces is that there is no convective mass transfer through the free surface. The stress boundary condition, that the forces acting on the fluid in contact at the free surface are in equilibrium, is

$$-p\mathbf{n} + \left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\mathbf{n} = \mathrm{C_a}^{-1}\kappa\mathbf{n} - P_a\mathbf{n} \tag{2}$$

where $\mathbf{n}$ the unit vector normal to the interface, P_a represents the air pressure of P_2 or P_1, $\kappa=\kappa^* H$ is the dimensionless interface curvature and $C_a=U\mu/\sigma$, where σ is surface tension.

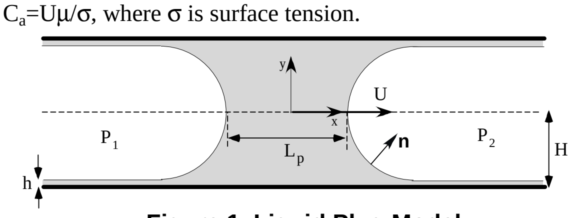

Figure 1. Liquid Plug Model

RESULTS

Trailing Film Thickness

Figure 2 shows the trailing film thickness, h_1, at Ca=0.05 for 0<Re<100, and for 4 values of Lp. The dash line represents the result for a semi-infinite bubble, i.e Lp=inf. For a fixed value of Lp, h_1 exhibits a local minimum as Re increases 0<Re<100. As Lp decreases the minimum value of h_1 decreases and the corresponding Re at the

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minimum increases. For Lp>1 h_1 shows similar behavior as the semi-infinite bubble.

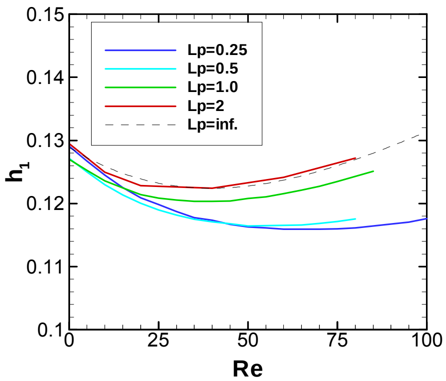

Figure 2. Trailing Film Thickness

Flow and Pressure Fields

Figure 3 shows streamlines and pressure fields inside the liquid plug at Re=0, 50 and 100, and at Ca=0.05 and L_p=0.25. The recirculation inside the plug is almost symmetric at Re=0. As Re increases the recirculation is increasingly skewed toward the back meniscus. At the far right of the domain, the liquid pressure equals the gas pressure, P_2, which we defined as the reference pressure and equals to zero. At the far left of the domain, the liquid pressure equals the gas pressure, P_1, which increases as Re increases. At the front meniscus the interface forms a capillary wave, where the pressure has a minimum. The pressure at the capillary wave varies corresponding to the film thickness variation. The amplitude of the pressure at the capillary wave increases as Re increases, while the wavelength is slightly shorter as Re increases.

Figure 4 shows the pressure difference $\Delta P=P_1-P_2$, divided by the plug length Lp, at Ca=0.05 for 0<Re<100. In a limit Lp=inf. the ratio $\Delta P/Lp$ approaches the Poiseuille flow limit of $3(1-h_1)$. Lp=2 curve lies above this limit and shows a slope of 0.0312. As Lp decreases $\Delta P/Lp$ increases for fixed Re and the slope increases.

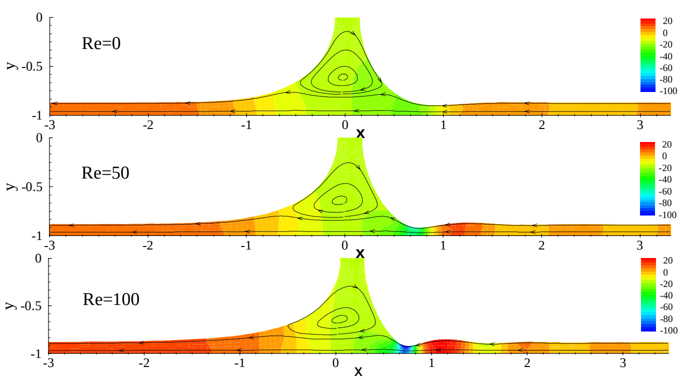

Figure 3. Streamlines and Pressure Fields.

Wall Shear Stress

Figure 5 shows the dimensionless wall shear stress, $\tau=\tau^*/(\mu U/H)$ distribution at Ca=0.05 and Lp=0.25. At the capillary wave, τ varies corresponding to the film thickness. As Re increases the amplitude of this wave is larger and the period is shorter. The τ shows a sharp peak where the film thickness is minimized.

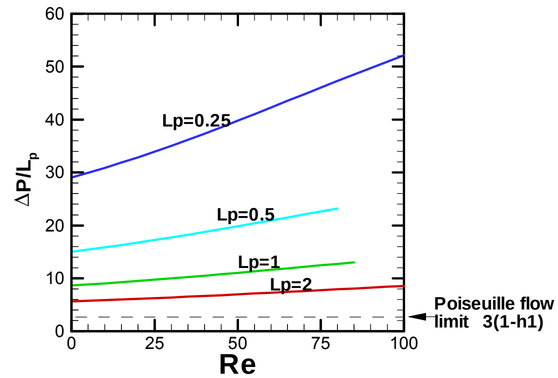

Figure 4. Driving Pressure drop divided by the plug length.

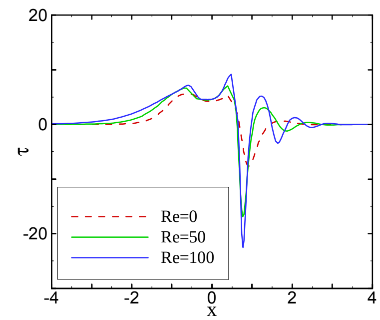

Figure 5. Wall Shear Stress

DISCUSSION

For medical purposes the trailing film thickness, h_1, is an important factor for the liquid. The pressure difference between both air-phases, ΔP, is also important in order to control the plug speed in a desired range. In this study we have shown that h_1 and $\Delta P/Lp$ vary with Re but also the plug length, Lp. The dimensional value, $\Delta P^*/Lp^*$ will be a function of Re and Re^2. The Re^2 term dominates as Lp decreases. The change of the recirculation inside the plug will affect mass transport processes. This is important for bulk convection of surfactant or other species and its concentration in the trailing film surface. The wall shear stress shows a sharp peak in the capillary wave. This peak wall shear stress increases with Re and reaches physiologically important values.

ACKNOWEDGEMENT

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REFERENCES

1. Cassidy, K. J., Bull, J. L., Glucksberg, M. R., Dawson, C. A., Haworth, S. T., Hirschl, R. Gavriely, N., and Grotberg, J. B., 2001, "A rat lung model of instilled liquid transport in the pulmonary airways." J. Appl. Physiol., 90, 1955.
2. Howell, P. D., Waters, S. L. and Grotberg, J. B., 2000, "The propagation of a liquid bolus along a liquid-lined flexible tube." J. Fluid Mech., 406, 309.
3. Waters, S. L. and Grotberg, J. B., 2002, "The propagation of a surfactant laden liquid plug in a capillary tube", Phys. Fluids, 14, 471.
4. Heil, M., 2001, "Finite Reynolds number effects in the Bretherton problem." Phys. Fluids, 13, 2517.