A NEW ANALYTICAL APPROACH TO EVALUATE THE VISCOELASTIC PROPERTIES OF THE GOAT MEDIAL COLLATERAL LIGAMENT USING THE QUASI-LINEAR VISCOELASTIC THEORY

Steven D. Abramowitch and Savio L-Y. Woo

Musculoskeletal Research Center
Department of Orthopaedic Surgery
University of Pittsburgh
Pittsburgh, PA

INTRODUCTION

The quasi-linear viscoelastic (QLV) theory proposed by Fung (1972) has been used extensively to model the viscoelastic behavior of many soft tissues [1]. Theoretically, the parameters describing the instantaneous elastic response and spectrum of relaxation are obtained following a step change in strain. As this is physically impossible, previous investigators have developed alternative approaches that include various assumptions and approximations to gain these parameters from experiments with finite ramp times [2]. The purpose of this work was to present a new analytical approach to find solutions for describing the instantaneous elastic response and reduced relaxation function of QLV theory using experimental data with finite ramp times. The analysis further utilized a non-parametric bootstrapping approach to assess parameter variability.

EXPERIMENTAL APPROACH

The femur-MCL-tibia complexes (FMTC) of six goat knees that had undergone a sham operation were used. With a laser micrometer system, the cross-sectional areas of the MCL were measured individually. Each FMTC was then mounted onto customized clamps in an Instron™ testing machine and tested in a saline bath at 32° C. Following an established testing protocol [3], a stress relaxation test was performed by elongating the FMTC to 3 mm (≤5% strain) and held for 60 minutes. Strain at the midsubstance was measured using a Motion Analysis™ system to track reflective tape markers placed on the ligament. The average strain rate during ramping was 0.15 ± 0.06 %/sec (mean ± SD) and the time until the peak load was 18.4 sec. The corresponding loads were collected at a rate of 5 Hz. For the purposes of this abstract, only every 5th data point was used in the analysis.

THEORETICAL APPROACH

The stress σ(t) in response to a general strain history can be written as the convolution integral of the elastic response $\sigma^e(\varepsilon)$ and the reduced relaxation function G(t) [1]:

$$\sigma(t)=\int_{-\infty}^{t} G(t-\tau)\frac{\partial\sigma^{e}(\varepsilon)}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial\tau}\partial\tau \qquad \{1\}$$

The stress during the ramping phase of a stress relaxation experiment from $0<t<t_0$ with strain rate α is

$$\sigma(t)=\frac{AB\alpha}{1+C\ln(\frac{\tau_2}{\tau_1})}\int_{0}^{t}\{1+\mathrm{C}(\mathrm{E}_1[\frac{(\mathrm{t}-\tau)}{\tau_2}]-\mathrm{E}_1[\frac{(\mathrm{t}-\tau)}{\tau_1}])\}e^{\mathrm{B}\alpha\tau}\partial\tau \qquad \{2\}$$

While, the subsequent stress relaxation for $t_0<t$ at a constant strain of α^{*} t_0 can be written as:

$$\sigma(t)=\frac{AB\alpha}{1+C\ln(\frac{\tau_2}{\tau_1})}\int_{0}^{t_0}\{1+\mathrm{C}(\mathrm{E}_1[\frac{(\mathrm{t}-\tau)}{\tau_2}]-\mathrm{E}_1[\frac{(\mathrm{t}-\tau)}{\tau_1}])\}e^{\mathrm{B}\alpha\tau}\partial\tau \qquad \{3\}$$

where $E_1[y]$ is the exponential integral. Since equations 2 and 3 consist of the same set of parameters, an objective function can be defined by adding the sum of square differences between equation 2 and the ramping data together with those between equation 3 and relaxation data. By minimizing this objective function with a nonlinear optimization algorithm, the ramping and relaxation data is simultaneously curve-fit to obtain parameters A, B, C, τ_1, and τ_2. As a high correlation between parameters A and τ_1 was observed, parameter A was held fixed. Its value was determined from curve-fitting the ramping portion of the experimental data with an exponential function:

$$\sigma^e(\varepsilon) = A(e^{B\varepsilon} - 1) \quad \{4\}$$

NUMERICAL APPROACH

Using a modified Levenberg-Marquardt algorithm, the parameters describing the average of all 6 data sets were obtained using the theoretical approach described in the previous paragraph (Figure 1). As residual plots displaying the error between observed and predicted stresses were non-Gaussian, a non-parametric bootstrapping analysis was then performed to determine parameter variability that may result from systematic deviations between the model and the experimental data, experimental noise, and numerical instabilities [4]. Residual plots for each specimen were curve-fit with a polynomial function to obtain a pool of curves representing systematic error (Figure 2). Taking the difference between the systematic error curves and the residuals allowed for a random noise distribution for each specimen to be obtained. Finally, a randomly selected systematic error curve and random error distribution were added to the predicted stresses of the averaged experimental data. This created a set of data whose systematic error and random noise distribution were representative of those observed experimentally. Parameters A, B, C, τ_1, and τ_2 for this new data set were obtained using the theoretical approach described above. One hundred new data sets were generated using this approach and 95% confidence intervals for each parameter were obtained.

RESULTS

Specimens stress relaxed by 32 ± 11% at 60 minutes following ramping to a peak stress of 15.3 ± 5.40 MPa. The relaxation was considered to have reached a plateau after 60 minutes as the slope for stress relaxation was -0.02 ± 0.02 %/min. The QLV model fit the data for each specimen well with R^2 values measuring 0.993 ± 0.006. Bootstrapping analysis showed that none of the parameters displayed large variability as measured by 95% confidence intervals (Table 1). Parameter A displayed the greatest variability with respect to its median (approximately 1%). Parameters B, C, τ_1, and τ_2 displayed a variability of less than 0.7% with respect to their medians.

DISCUSSION

This study presented a new approach to determine the parameters describing the instantaneous elastic response (A and B) and reduced relaxation function (C, τ_1, and τ_2) of QLV theory. By using experimental data with finite ramp times, this approach provides a simultaneous and, more importantly, direct fit to the experimental data of the ramping and relaxation portions of a stress relaxation test. Thus, it is not necessary to normalize the relaxation data by the peak stress at t_0. The minimal variability of all 5 parameters suggests that this approach is neither numerically unstable nor sensitive to systematic deviations between the QLV model and the experimental data. The data obtained for A, B, C, τ_1, and τ_2 are similar to those reported previously for the canine MCL in spite of the fact that the strain rate used in the present study was 2 orders of magnitude slower [5]. Thus, accurate estimates of A, B, C, τ_1, and τ_2 may be obtained using this approach regardless of the experimental ramp time.

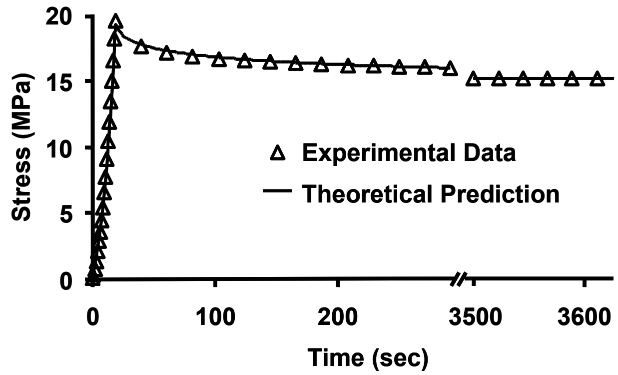

Figure 1: A typical curve-fit of the experimental data.

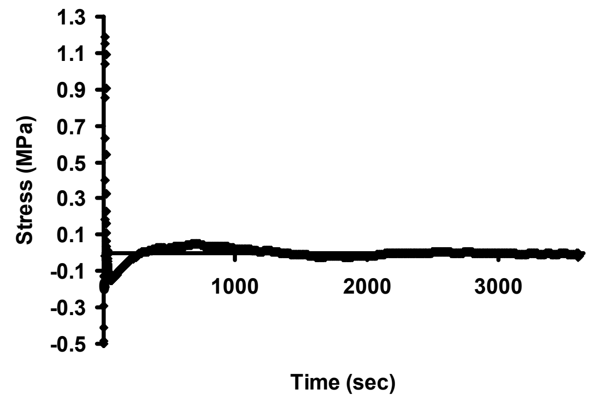

Figure 2: A typical residual plot demonstrating systematic deviations between the QLV model prediction and experimental data.

A	B	C	τ_1 (sec)	τ_2 (sec)
5.77	47.2	0.0721	0.62	1469
6.00	48.2	0.0724	0.63	1488

Table 1: 95% confidence intervals for the parameters describing the instantaneous elastic response (A and B) and the reduced relaxation function (C, τ_1, and τ_2).

REFERENCES

1. Fung, 1972, Biomechanics, 181-208
2. Kwan et al., 1993, J. Biomech, 26: 447-52
3. Scheffler et al., 2001, Ann Biomed Eng, 29: 173-80
4. Yin et al., 1986, J. Biomech, 19:27-37
5. Woo et al., 1983, JBME, 109:68-71

ACKNOWLEDGEMENTS

The support of NIH grant AR 41820 is gratefully acknowledged.