A CONTINUUM TRANSVERSELY ISOTROPIC VISCOHYPERELASTIC CONSTITUTIVE LAW APPLICATION TO THE MODELING OF THE HUMAN ACL

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INTRODUCTION

The main characteristics of biological soft connective tissues is that they sustain large deformations and displacements, have a highly nonlinear behaviour and possess strongly anisotropic mechanical properties [1]. Moreover, their behavior is known to be viscoelastic, and this is especially relevant as the loading rates involved increase. It was shown that the shape of the stress-strain curves at which traction tests are performed is affected by the loading rate [2-3]. Although numerous viscoelastic models have been proposed in the past [4-5], their domain of validity is restricted to low strain rates (0.06-0.75 %.s^{-1}) [6] and give inaccurate results for higher strain rates (up to 10 %.s^{-1}) [7]. The continuum constitutive framework developed by Pioletti et al. [8] had the merit to encompass strain rate effects by using the rate of deformation as an explicit variable. However, a serious shortcoming of this constitutive law was the assumption of isotropy. In fact, due to their fibrous structure (collagen fibers embedded in a highly compliant solid matrix), modeling ligaments as anisotropic structures is indeed a basic necessary requirement [9-10]. The first objective of the present study is therefore to extend the work from Pioletti et al. [8] to the transversely isotropic case (the simplest form of anisotropy) by combining the constitutive framework of Noll [11] and the theory of continuum fiber-reinforced composite of Spencer [10,12]. The second objective is to demonstrate the suitability of the constitutive framework by proposing an original constitutive law and identifying the material parameters with experimental data for the ACL [8].

METHODS

The constitutive framework presented is based on the postulate that there exists a Helmholtz free energy function y, isotropic function of its arguments [13]. The energy function y is a function of $\mathbf{C}$, $\mathbf{N_0}=\mathbf{n_0}\otimes\mathbf{n_0}$ ($\mathbf{n_0}$ is a unit vector representing the local orientation of the fibers) and a set of tensorial thermodynamic variables $\mathbf{A}_k$. $\mathbf{C} = \mathbf{F}^T.\mathbf{F}$ is the right Cauchy-Green deformation tensor and $\mathbf{F}$ is the deformation gradient. It is assumed that the equilibrium state of the viscoelastic solid at fixed $\mathbf{F}$ as $t \to \infty$ derives from a strain energy function y^e.

For isothermal processes, the First and Second Principles of Thermodynamics are expressed by the Clausius-Duhem inequality [11] which, combined with the existence of y^e leads to:

$$\Xi_{\text{int}} = \frac{1}{2}\left(\mathbf{S} - 2\frac{\partial y^e}{\partial \mathbf{C}}\right) : \dot{\mathbf{C}} \geq 0 \quad \forall\ \mathbf{C}, \dot{\mathbf{C}} \tag{1}$$

where Ξ_{int} is the local entropy production, $\dot{\mathbf{C}}$ is the rate of deformation and $\mathbf{S}$ is the total second Piola-Kirchhoff stress tensor. It is assumed that the local entropy is produced by viscous effects which derive from a viscous potential y^v. Following the standard Coleman-Noll procedure [14] to ensure that the dissipation is positive for all admissible processes (arbitrary choices of $\dot{\mathbf{C}}$), we obtain:

$$\Xi_{\text{int}} = \frac{\partial y^v}{\partial \dot{\mathbf{C}}} : \dot{\mathbf{C}} = \frac{1}{2}\mathbf{S}^v : \dot{\mathbf{C}} = \left(\mathbf{S} - \mathbf{S}^e\right) : \dot{\mathbf{C}} \geq 0 \quad \forall\ \mathbf{C}, \dot{\mathbf{C}} \tag{2}$$

where $\mathbf{S}^e$ and $\mathbf{S}^v$, denote respectively the fictitious elastic and viscous second Piola-Kirchhoff stress tensors. A set of 17 invariants are necessary to form the *irreducible integrity bases* of the tensors $\mathbf{C}$, $\dot{\mathbf{C}}$ and $\mathbf{N_0}$ [13]. In other words, it must exist a function $\bar{y}^e$ such that $y^e(\mathbf{C}, \mathbf{N_0}) = \bar{y}^e(\{I_a(\mathbf{C}, \mathbf{N_0})\}_{a=1..5}$ and $\bar{y}^v$ such that $y^v(\mathbf{C}, \dot{\mathbf{C}}, \mathbf{N_0}) = \bar{y}^v(\{J_a(\mathbf{C}, \dot{\mathbf{C}}, \mathbf{N_0})\}_{1..12}$ where I_a and J_a are tensorial invariants (see [13] for further details).

Based on structural and physiological properties of ligaments [9], a particular Helmholtz free energy function $\bar{y} = \bar{y}^e + \bar{y}^v$ for the ACL ise proposed:

$$\bar{y}^e = \begin{cases} c_1(I_1 - 3) & \text{if } I_4 \leq 1 \\ c_1(I_1 - 3) + \dfrac{c_2}{2c_3}[e^{c_3(I_4-1)^2} - 1] & \text{if } I_4 > 1 \end{cases} \tag{3}$$

$$\bar{y}^v = \begin{cases} 0 & \text{if} \quad I_4 \le 1 \\ h_1 J_2 \left(I_1 - 3\right) + h_2 J_5 \left(I_4 - 1\right)^2 & \text{if} \quad I_4 > 1 \end{cases} \tag{4}$$

c_1, c_2, c_3, h_1 and h_2 are material parameters which warrant convexity of the elastic and viscous potentials, provided that they are all positive and that incompressibility has been assumed. The function y^e and y^v decouple the matrix and fiber contributions.

The fictitious Lagrangean elastic and viscous stress tensors are:

$$\mathbf{S}^e = \begin{cases} 2c_1\mathbf{1} + p\mathbf{C}^{-1} & \text{if} \quad I_4 \le 1 \\ 2\left[c_1\mathbf{1} + c_2 e^{c_3 (I_4 - 1)^2} (I_4 - 1)\mathbf{N_0}\right] + p\mathbf{C}^{-1} & \text{if} \quad I_4 > 1 \end{cases} \tag{5}$$

$$\mathbf{S}^v = \begin{cases} 2\left[h_1 (I_1 - 3)\dot{\mathbf{C}}\right] & \text{if} \quad I_4 \le 1 \\ 2\left[h_1 (I_1 - 3)\dot{\mathbf{C}} + h_2 (I_4 - 1)^2 \; \text{¡}_{\mathbf{n_0}\dot{\mathbf{C}}}\right] & \text{if} \quad I_4 > 1 \end{cases} \tag{6}$$

where $\text{¡}_{\mathbf{n_0}\dot{\mathbf{C}}} := \mathbf{n_0} \otimes \dot{\mathbf{C}}.\mathbf{n_0} + \mathbf{n_0}.\dot{\mathbf{C}} \otimes \mathbf{n_0}$ and p is the pressure.

Through a nonlinear optimization procedure, the material parameters of the transversely isotropic hyperviscoelastic constitutive law were identified with experimental data related to tensile tests (in the natural fiber orientation) of the human ACL at different loading rates [8]:c_1=1 MPa, c_2=2.31±0.21 MPa, c_3=7.97±0.36, h_1=54.38±0.02 MPa.s , h_2=0.13±0.00 MPa.s (R^2=0.999). The response of the isotropic elastic matrix is governed by c_1, the value of which has been fixed [9].

RESULTS

The proposed constitutive law is able to fit very closely the experimental results and highlights the typical strain rate effects (Figure1).

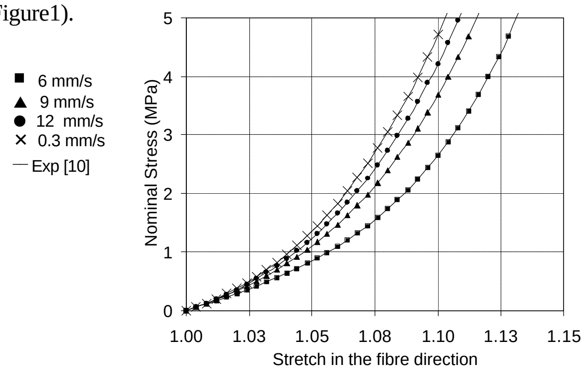

Figure 1. Stress-stretch curves for tensile tests of the ACL. Fitting of the constitutive law to experimental test data.

DISCUSSION

A constitutive framework encompassing simultaneously transverse isotropic hyperelasticity and viscosity with arbitrary strain rate dependency has been developed (for sake of brevity, numerous developments have not been presented). The general constitutive law is thermodynamically admissible a priori and is valid for arbitrary kinematics. It is particularly suitable for modeling the mechanical behavior of biological soft tissues at high strain rates. This is of high relevance for dynamic analyses of human occupants in car crash simulations (finite element analyses) and for situations where dynamic loads are significant (sport injury, etc).

The particular proposed Helmholtz free energy function, while encompassing essential features of the ligaments (nonlinear behavior, stiffening in extension and high compliance in compression along the fiber direction, incompressibility, finite strain, anisotropic viscous response and strain rate effects), was capable of fitting accurately the analytical-experimental curves from Pioletti et al. [8]. The pure elastic response is described by only three parameters while the viscous response can be accurately described by one (although two parameters have been used in this study). The general framework will prove useful to test different elastic and viscous potentials as soon as new relevant experimental data will become available.

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