A NOVEL STRAIN ENERGY FUNCTION FOR ARTERIES ACCOUNTING FOR COMPOSITION AND STRUCTURE

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INTRODUCTION

The artery's highly non-linear incremental elastic modulus and its anisotropy are two of its most prominent mechanical features. While many mathematical models of the arterial wall under totally passivated VSM exist [1, 2], most neglect the composition and structure of the wall matrix. We thus propose such a description within the classical framework of finite soft tissue mechanics [2] using a strain energy function (SEF), which makes use of histomorphometrical data and parameters that bear structural or mechanical significance, such as principal fiber orientations or elastic moduli and is based on one proposed by Holzapfel et al. [3].

METHODES

Theoretical Considerations

We assume, that the arterial wall is incompressible. Further, ring segments cut open are assumed to be in a stress-free configuration [4], the zero stress state (ZSS), which we use as a reference configuration. For a given SEF Ψ , using the internal radius r_i and the external radius r_o of the artery when axially stretched and inflated we can determine the experimentally accessible quantities lumen pressure *P*

$$P = \int_{r_i}^{r_o} \left(\lambda_{\theta}^2 \frac{\partial \Psi}{\partial E_{\theta}} - \lambda_r^2 \frac{\partial \Psi}{\partial E_r} \right) \frac{dr}{r}$$
(1)

and axial force F_z via local radial stress

$$\sigma_r = \int_{r_l}^r \left(\lambda_\theta^2 \frac{\partial \Psi}{\partial E_\theta} - \lambda_r^2 \frac{\partial \Psi}{\partial E_r} \right) \frac{dr}{r} - P \tag{2}$$

$$F_{z} = \int_{r_{i}}^{r_{o}} \left(\sigma_{r} + \lambda_{z}^{2} \frac{\partial \Psi}{\partial E_{z}} - \lambda_{r}^{2} \frac{\partial \Psi}{\partial E_{r}} \right) 2\pi r dr - P \pi r_{i}^{2}$$
(3)

where E_{θ} , E_z , and E_r are Green strains and λ_{θ} , λ_z , and λ_r are the stretch ratios relative to the ZZS.

Novel Strain Energy Function

As proposed by Holzapfel et al. [5] we split the SEF Ψ into an isotropic (Ψ_{iso}) and an anisotropic (Ψ_{aniso}) component assuming that their effects are purely additive. We propose weighting each component with the histomorphometrically determined cross-section area fractions of elastin (f_{elast}) and collagen (f_{coll}) for the isotropic and anisotropic parts, respectively:

$$\Psi = f_{elast} \Psi_{iso} + f_{coll} \Psi_{aniso} \tag{4}$$

For the isotropic elastin SEF we assume

$$\Psi_{iso} = c_{elast} (I_1 - 3)^2$$
 (5)

where c_{elast} is an elastic modulus and I_1 is the first invariant of the Cauchy-Green strain tensor.

From histology [6] it is known that collagen fibers assume a wavy configuration when the artery is unloaded. When the artery is submitted to axial elongation and lumen pressure, the fibers straighten out and begin to bear tensile loads. Thus, we assume that the engagement strain of the individual collagen fibers is distributed loglogistically and can be described by a probability distribution function (PDF) with two parameters, k and b. Folding the engagement of a single fiber with the PDF we obtain an effective strain for the entire fiber ensemble

$$\varepsilon_{effective}(\varepsilon) = \int_{-\infty}^{\infty} x \cdot \Theta[x] \cdot PDF[\varepsilon - x, b, k] dx$$
(6)

with $\Theta[x]$ the unit step function and ε the actual local wall strain in fiber direction affecting the fiber ensemble. The fibers are assumed to be oriented at a fixed angle α to the circumferential direction in the θ -z plane [3]. Thus we propose for the anisotropic SEF:

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$$\Psi_{aniso} = c_{coll} \left(\varepsilon_{effective} \left[\sqrt{I_4 - \log[I_4]} - 1, \alpha, k, b \right] + 1 \right)^2$$
(7)

where I_4 is another invariant of the Cauchy-Green strain tensor in respect to the fiber orientation [7].

Experimental Data

As described in detail elsewhere [8], pressure-radius (P-r) and pressure-axial force $(P-F_z)$ measurements of rat carotids were performed in the presence of papaverine which fully passivated the VSM. Data was obtained at *in vivo* axial stretch, 115% *in vivo* axial stretch, and 130% *in vivo* axial stretch. Ring segments where extracted from the carotids and cut open to provide opening angles and radii of the artery's ZSS used as reference state in the model. Collagen and elastin content was determined histomorphometrically.

The model data was fitted to the experimental data to determine the unknown parameters of the SEF by minimizing

$$\Phi = \frac{1}{2} \frac{1}{mn} \sum_{i,j}^{m,n} \left(\frac{r_{ij}^{\text{mod}} - r_{ij}^{\text{exp}}}{\sigma_{ij}^{r}} \right)^{2} + \frac{1}{2} \frac{1}{mn} \sum_{i,j}^{m,n} \left(\frac{F_{ij}^{\text{mod}} - F_{ij}^{\text{exp}}}{\sigma_{ij}^{F}} \right)^{2}$$
(8)

Indices *i* and *j* denote pressure and axial elongation at which the corresponding radius *r* and axial force *F* measurements were obtained, respectively. *m* is the number of different pressures and *n* the number of different axial stretch ratios. σ is the standard deviation of the experimental mean value of radius or force and is used as a weighting factor. On the forces *F* and radii *r* the indices *mod* and *exp* are used to identify the model and experimental values, respectively.

The many parameters (four) left open in the description of the anisotropic part of the SEF left much freedom and we thus decided to fix the elastic modulus for the collagen fibers to c_{coll} =200MPa, equivalent to a physiologically reasonable Young modulus of 400MPa [9] at the onset of fiber engagement.

RESULTS

The novel SEF does a good job describing the experimental data (Figure 1). The obtained parameters were c_{elast} =48.4kPa, b=14.8, k=0.406, and α =33.1° for fixed f_{elast} =0.3, f_{coll} =0.2, and c_{coll} =200MPa.



Figure 1. Experimental (dots) and novel SEF (line) pressureradius and pressure-axial force plots of rat carotid arteries. Bars denote experimental standard deviations.

DISCUSSION

Aside from the good description of the experimental data, it is important to notice that the fitted parameters, all bearing some sort of structural significance, are in good agreement with literature. When we calculate the Young modulus for the isotropic non-linear elastin SEF, we find that the values (284kPa at 40mmHg, before significant collagen fiber engagement) are within the range of 100kPa to 1MPa reported in literature [9]. Collagen fibers are reported have an organized structural appearance at pressures above 80mmHg. The parameters describing the fiber engagement, k and b, show that 44.1% of the fibers would be engaged and straightened by 80mmHg and 84.1% by 90mmHg. At the end of the experiment, 99.1% of the fibers are engaged in load bearing, according to the model. Finally, the collagen fibers have been said to form a "uniform helix of small pitch" [6]. The angle of 33.1° obtained from the fit might or might not fulfill these rather vague observations. In any case, the direction of the collagen fibers is probably not entirely uniform and a more advanced SEF might include an angle distribution.

We conclude that, even though many SEF describing the arterial wall mechanics exist, most have been excessively phenomenological and few have made direct use of histological information such as in [3]. The here presented SEF succeeds in describing arteries and while all its parameters bear some structural meaning providing possible predictive power for pathologies related to arterial structure.

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