FRACTIONAL-ORDER VISCOELASTICITY APPLIED TO HEART VALVE TISSUES

Evelytn O. Carew (1), Todd C. Doehring (1), J. Edward Barber (1), Alan D. Freed (2), Ivan Vesely (1)

(1) Lerner Research Institute
Department of Biomedical Engineering
The Cleveland Clinic Foundation
Cleveland, Ohio

(2) Polymers Branch
NASA Glenn Research Center at Lewis Field
Brookpark, Ohio

INTRODUCTION

There is an ongoing need to develop efficient, high fidelity material models to simulate the stress (or strain) response of soft biological materials. Fractional Calculus is being used to develop fractional-order viscoelastic (FOV) constitutive equations for applications to soft biological tissues. Fractional calculus is concerned with the development and application of differential equations of non-integral order b ($0 < b < 1$), and the methods for their solution [1, 2]. The attraction in using a constitutive description based on fractional calculus for modeling soft tissues is their potentially superior accuracy, and the possibility of correlating the hierarchical structure of biological tissues to the fractional order b [3]. Fractional calculus formulations have recently been applied to a number of biological tissues and processes [3 - 5]. In a continuing effort to assess the ability of FOV to represent soft tissue behavior, we formulated a one-dimensional version of FOV - "quasilinear fractional-order viscoelasticity" (QLFOV), and applied it to porcine aortic valve (PAV) valve tissues. A PAV cusp is a tri-layered, inhomogeneous, anisotropic, nonlinear viscoelastic material and provides a non-trivial test of the capabilities of any potential soft tissue model. Our objectives were to: (i) evaluate QLFOV material parameters for PAV tissues from stress-relaxation experiments done at a reasonably high strain-rate, and (ii) assess the predictive ability of QLFOV by simulating the stress response of the heart valve tissues to a saw-tooth strain history.

MATERIALS AND METHODS

The quasi-linear form of the constitutive equation (CE) for one dimensional fractional-order viscoelasticity has been derived from the constitutive equation for the standard, linear, fractional-order viscoelastic solid [6] and can be written as:

$$T(t) = \int_{0+}^{t} G_b(t-s)\,\frac{dT^e[e(s)]}{de}\,\dot{e}(s)\,ds \qquad (1)$$

where $T(t)$ is the stress (or tension) at time t due to a strain history $e(s)$, $0 < s$ £ t, and $G_b(t)$ and $T^e(t)$ are material functions: $G_b(t)$ is the normalized relaxation modulus (NRM) and $T^e(t)$ is the elastic response (ER). The NRM is given by:

$$G_b(t) = a^b + (1-a^b)E_b[-(t/l)^b], \quad a = l/r \qquad (2)$$

where b ($0 < b < 1$) is the fractional order of evolution, λ (> 0) is the characteristic time for stress relaxation (relaxation time), ρ (>λ) is the characteristic time for creep (retardation time), and $E_b(z) = E_{b,1}(z) = E(b,1,z)$ is the Mittag-Leffler function [1]. We note that $E_b(0+) = 1$; thus $G_b(0+) = 1$, and $G_b(t)$ is a normalized function of time. The role of the Mittag-Leffler function in fractional calculus is similar to that of the exponential function in traditional calculus; in fact they are related through $E_{1,1}(t) = e^t$. The ER is derived from the measured stress-strain response $T^m(e)$, which is represented by:

$$T^m(e) = \begin{cases} a(e^{be} - 1) + ce & e \le e_T \\ d(e - e_T)^3 + e(e - e_T)^2 + f(e - e_T) + g & e > e_T \end{cases} \qquad (3)$$

The piecewise exponential-cubic function (3) can accurately represent the non-linear elastic behavior of heart valve tissues that typically have a long toe region [7]. In (3), $a, b, c, d, e,$ and f are fitted parameters and ε_T is a specified transition strain. As in quasilinear viscoelastic (QLV) theory, the material functions $G_b(t)$ and $T^e(e)$ are to be obtained from step-strain experiments in order to justify the strain-time convolution manifested in (1). Since such experiments are impractical, alternative experiments at sufficiently high speed usually suffice, although special methods for accurate parameter extraction are required.

Parameter Estimation

To approximate the true material parameters from practical finite strain-rate experiments, we follow the procedure given in [8]. Briefly, the ER is represented by a scaled-up version of the measured stress-strain response (3), the scale-up factor being an additional parameter that enforces equality of stress at the end of loading and the start of the stress relaxation of the hypothetical infinite strain-rate experiment. Both loading and relaxation data are fitted separately to (3) and (2), respectively. The CE is then imposed at the start and end of the relaxation experiment resulting in two equations. We solved the equations as a system for the scale-up factor, and in order to keep the system linear, α.

Materials

PAVs were excised from fresh hearts obtained from a local abattoir. Rectangular strips of dimensions 10 mm x 5 mm (n = 3) were cut from the cusps in the circumferential direction (Fig. 1).

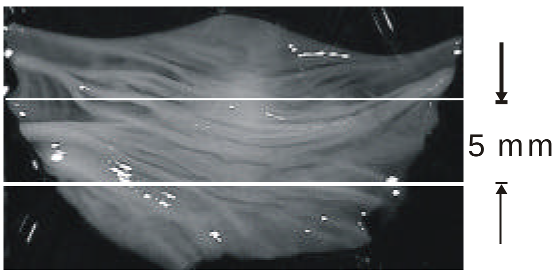

Figure 1: Porcine aortic valve cusp showing a typical circumferentially oriented strip used in tensile testing.

Experimental Tests

Tensile testing was performed with an Instron 8511 servo hydraulic testing machine (Plus series, Instron, Canton, MA), using a 5.0 lb load cell (Sensotec, Columbus, OH). All tests were conducted in a bath of Hanks physiologic saline solution at 37°C. Each specimen was held in sandpaper-lined plastic grips that were inserted between the actuator and the load cell of the testing system. Prior to tensile testing, specimens were subjected to a novel preconditioning protocol designed to generate repeatable hysteresis loops and stress relaxation curves simultaneously. Immediately after the preconditioning, each specimen was loaded at 40 mm/s to a pre-determined displacement equivalent to a load of 600 g, and maintained at that displacement for 1000 s (stress relaxation). The specimen was then unloaded at 4 mm/s to zero load. This initial stress relaxation test was used for specimen characterization according to the method described above. Immediately after the stress relaxation test, the specimen was subjected to low amplitude (5% strain) saw-tooth straining at 4 mm/s, the saw-tooth strain history being restricted to the linear range of the specimen's measured stress-strain data. All loads were converted to tension through appropriate scaling.

Simulating the saw-tooth experiment

With the specimen characterized from the initial relaxation test (material parameters known) and the experimental saw-tooth strain history as input, the stress response was predicted using (1), and compared to the experimental data.

Error Quantification

Least squares relative error measurements were used to quantify both global and local (stress peaks and valleys) errors in the prediction of the stress.

RESULTS

We found that both initial relaxation rate and overall relaxation are governed predominantly by b and a. For PAV circumferential strips, the material parameters were (mean ± SEM): $b = 0.36 \pm 0.02$, $a = 0.37 \pm 0.04$, and $l = 0.5 \pm 0.01$. QLFOV was reasonably accurate in predicting the saw-tooth stress response of PAV specimens: peak error = 8.4% ± 2.2%, valley error = 7.5% ± 0.7%, and global error = 9.2% ± 1.7%; (Fig. 2).

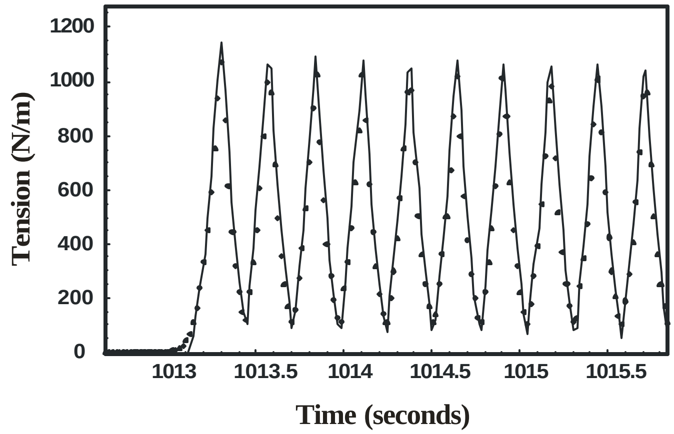

Figure 2: Comparison of QLFOV prediction (solid line) to experimental data (points).

CONCLUSION

We have demonstrated that quasi-linear fractional order viscoelastic theory can be used to model the stress response of porcine aortic valves with good fidelity. Work is now in progress to apply QLFOV to the 1D response of other biological tissues and to implement FOV in 3D and for finite deformations.

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