THE DYNAMIC RESPONSE OF CEREBRAL BLOOD FLOW TO CHANGES IN PRESSURE

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INTRODUCTION

The Circle of Willis (COW) is a ring-like network of blood vessels located at the base of the brain, below the hypothalamus. Its main function, is to act as a distribution centre for the oxygen-rich blood flowing up from the heart to the rest of the cerebral mass. The COW is supplied by three main sources – the left and right internal carotid arteries (LICA and RICA) as well as the basilar artery (BA). If one of these supply arteries became occluded or stenosed, the region of cerebral tissue normally supplied by the artery can be starved of oxygen and become ischemic. The key importance of the COW lies in its distributive ability. Fresh blood being supplied by the other afferent arteries can be redirected to the region suffering a deficiency via the COW. The ability of the COW to fulfil its distributive function is dependent on the anatomical structure of the COW being complete, or balanced. A novel time-dependent, one-dimensional model of the COW has been developed so as to investigate the behavioural response of the COW to various physiological states.

The brain requires a constant supply of oxygenated blood in order to maintain normal function. Ideally, the pressure in the afferent vessels of the COW, e.g. right internal carotid, should be constant relative to the venous pressure [1]. However, this is not always the case. Standing up, sitting down, or, a surgical procedure such as a carotid endarterectomy where a change in pressure in the afferent vessel is induced, will cause a pressure change relative to the venous pressure. To maintain a constant supply of blood, the cerebro-vascular bed changes its resistance to flow either by vaso-dilation or vaso-constriction in order to maintain ideal flux [1]. This is known as autoregulation.

Hillen et al [2] developed a one-dimensional, steady-state model of the COW, where the relationship between the flux, resistance to flow and pressure was described by the Hagen-Poiseuille formula. With pulsatility and elasticity neglected, it was concluded that, firstly, the efferent fluxes were dependent upon the distribution of the efferent resistance, and secondly, there was a relationship between the flux and the anatomical structure of the COW. Ferrandez et al [3], developed a two-dimensional, time-dependent model of the COW coupled with cerebral autoregulation to investigate the effects of structural variation of the COW on the distribution of flux through the efferent vessels. For the future purpose of clinical use, a new one-dimensional, time-dependent model of the COW coupled with the autoregulatory capacity of the cerebro-vascular bed was developed. This one-dimensional model will require less computational time and also, have a much lower level of complexity in terms of usage.

This work seeks to investigate how a variation in the anatomical structure of the COW, together with the dynamic peripheral resistance response, will affect the distribution of efferent flux by the COW.

METHOD

Figure 1 shows a schematic representation of the one-dimensional COW used for the development of this model.

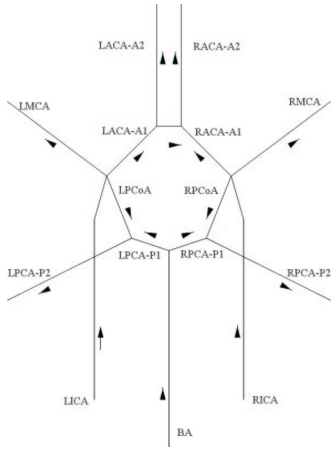

Figure 1: Schematic representation of the COW. The assumed direction of flow is indicated by the arrow heads.

By assuming that the vessel walls were rigid and that the blood flow was laminar and incompressible, the pressure-flux relation through a vessel can be described by the Hagen-Poiseuille formula for fluid

flow. Conservation of mass was also applied to each junction within the COW.

Autoregulation of Peripheral Resistance

The autoregulatory response of the resistance of the vascular bed was modelled using a feedback loop, previously described by Ferrandez et al [3], which consists of two main blocks, the Controller and the Plant (Figure 2).

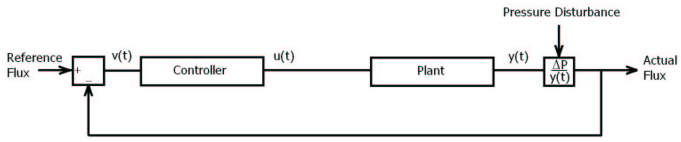

Figure 2: Figure of the Feedback loop

The controller used is a Proportional + Intergral (PI) controller. Mathematically, its behaviour can be described by the following equation :

$$u(t) = K_p v(t) + K_i \int v(t) dt$$

The plant describes the dynamic behaviour of the peripheral resistance. Mathematically, the dynamic response of the peripheral resistance can be described as :

$$\tau \frac{dy(t)}{dt} = u(t) - y(t)$$

where τ is the time delay of the response of the autoregulatory mechanism. This time delay is an important part of this model as it is important to recognise that the body is unable to produce instantaneous responses to changes in physiological conditions.

The physiological limits for vaso-dilation and vaso-constriction have also been included in the model [3]. This is an important part of the model as it must be recognised that the blood vessels cannot dilate or constrict infinitely. It is the inclusion of these limits that give a clear indication of how anatomical variations of the COW affect the distribution of flow to the efferent vessels.

RESULTS

A step pressure drop of 17 mmHg in introduced to the right internal carotid artery, simulating a sudden compression of the artery. Analyses of the results produced give an indication of how anatomical variations of the COW affect the distribution of efferent flux The results produced by this model were compared to the results reported by Ferrandez et al [3].It is seen that the configuration with the A1 segment of the left anterior cerebral artery (LACA-A1) absent produces the least ideal distribution of efferent flux. From Figure 3, where the ipsilateral non-dimensional fluxes are plotted as a function of time, it is seen that there is a 21% decrease in the flux through the right middle cerebral artery (MCA) and the anterior cerebral artery (RACA-A2). The posterior cerebral artery (RPCA-A2) only experiences negligible drop in flux. Figure 4 shows the contralateral efferent flux for the same configuration of the COW. It can be seen that there is a 21% decrease in the flux through the LACA-A2 and a 4% drop in the flux through the LMCA. The flux through the LPCA-P2 only experiences a slight drop in flux. Careful observation of the flux of the LACA-A2 reveals that after the initial drop, the flux does not recover to its initial value. When the LACA-A1 segment of the COW is absent, the peripheral resistance of that artery decreases, i.e. the vascular bed vaso-dilates so as to increase the flux through it. This flux will have to come from the ipsilateral side of the COW, which is also the side affected by the pressure drop. When the pressure drop occurs, the efferent flux through the LACA-A2 segment is unable to regain its full theoretical flux due to the cerebro-vascular bed reaching its physiological limits for dilation.

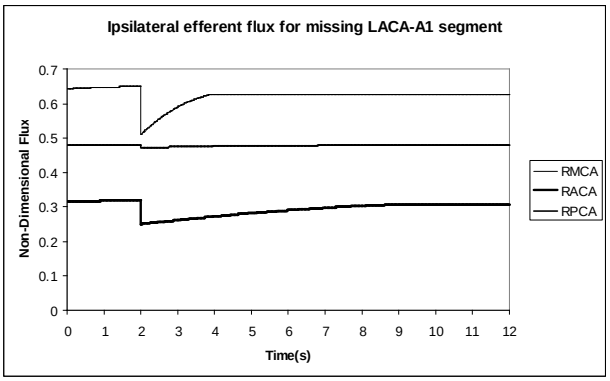

Figure 3 : Plot of ipsilateral efferent fluxes for a COW with missing A1 segment of the left anterior cerebral artery

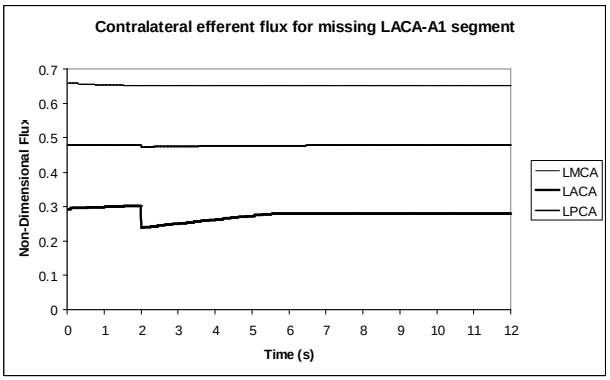

Figure 4 : Plot of contralateral efferent fluxes for a COW with missing A1 segment of the left anterior cerebral artery

CONCLUSION

This work has developed a model which investigates how different anatomical configurations of the COW influences the distribution of the flux through the efferent vessels when coupled with the dynamic responses of the cerebro-vascular bed. The further development of this one-dimensional model to one which is ready for clinical tests is then, relatively easy.

REFERENCES

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