IMAGE-BASED, DIRECT THREE DIMENSIONAL VASCULAR MODELING

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INTRODUCTION

The conversion of three-dimensional (3D) images to finite element (FE) models has important applications in medicine and biology, including studying processes such as blood flow, muscle and bone movement, and cellular transport. An important part of such a modeling process is the description of the spatial computational domain; typically, the spatial domain is broken up into small volume elements, known as a mesh. The objective of our research is to convert gridded 3D data into a compact, analytic, continuous (C0 or C1) Cubic Hermite surface description. This surface description can be converted to the desired FE mesh using standard modeling and meshing software. In addition, our surface description is a standard one for computer aided design packages, so that the resulting geometries can be easily manipulated. Though our method works for arbitrary surfaces, the specific application we will be concerned with is vascular modeling.

A major impediment to the more widespread clinical use of FE models is the time and user effort required to build them. The impediments to automatic vascular model generation include: the topological complexity of vascular beds, varying levels of importance of spatial detail, and patient specific geometry. The modeling approach described here is topology independent, easily and automatically refinable, and largely user independent, making it ideal for a variety of modeling applications.

Since in some domains (particularly the distal vasculature) spatial resolution is not as important (and imaging is not as reliable), the method has automatic means to spatially trim the 3D domain and extend it distally with a 1D model. This feature provides a mechanism to naturally couple downstream 1D models with the 3D domain.

MOTIVATION

Many current vascular models are based on an underlying cylindrical topology.[1,2] That is, models are constructed as a union of cylinders. For straight, non-branching vessels, this can be an adequate description. However, at a bifurcation, the topology is no longer cylindrical and such models break down. An example is shown in figure 1, where the discontinuity in surface derivative due to underlying topological assumptions, is clearly visible. In addition, current 2D based methods require an underlying path; though algorithms exist to automatically find a path, they necessarily work on the entire volume; the method described here, upon reaching a branch vessel under a critical radius, terminates the search. Thus, our method spares the computer time and potential algorithmic ambiguity that could result from processing the noisy distal vasculature.

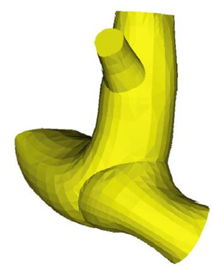

Figure 1

METHODS

A triangulated representation of the region of interest is required as input. This can be obtained from a thresholded volume image, output from a 3D level set simulation, or any other edge detection algorithm and would be followed by the Marching Cubes algorithm applied to the desired isosurface. Note that the triangulated input surface could be much larger than the desired surface sub-piece of interest since it may have hundreds of branch vessels and other structures (such as a noisy dense vascular bed, or a kidney or bone) topologically attached. The only topological input requirement is that these attached structures occur downstream of a vessel of cutoff radius, or upstream of the starting cut plane. Surface construction proceeds in a number of steps, each of which will be described below.

Step 1: Surface Isolation and Data Structure Generation

The user selects an inlet plane, thus selecting a surface from all surfaces returned by marching cubes, and severing the desired downstream vasculature from all upstream structures. The user also selects a patch radius (R_p) and vessel radius cutoff ($R_p < .5*R_c$). Note that this is the only user intervention required for the entire fitting process; however, if other cutting planes are needed (large branch vessels/ other inlets) these can be chosen as well.

The first step proceeds by covering the surface with patches of radius $\pi * R_c$. The procedure starts by determining the inlet curve (intersection of cutting plane with triangulated surface) and its length (L) and then dividing into L/($\pi * R_c$) vertex points, separated equally. Starting at each vertex point, a Fast Marching Method [3] is used to expand the solution to the Eikonal Equation (which gives distance from vertex to a point on the surface, as traveled along the surface) up to a distance R. Since the connectivity of triangles is not known a-priori, when patches are grown, their topologically connected neighbors are determined, and a connectivity structure for the surface is updated. New vertex points are chosen on the "free edges" (those which touch only one patch) of the current patch network. Patches are not expanded past cut planes. After a patch is added, its circumference is checked to make sure the patch is homeomorphic to a disk. If not, the vessel has a circumference less than $2 * \pi * R_c$, so that its "radius" is less than R_c; thus, the vessel should be trimmed by a cutting plane. After completion of this step, a structure like figure 2, but with larger patches, is generated; the result is an isolated trimmed surface whose connectivity and normal is known.

Step 2: Voronoi Diagram Generation

The inlet curve (intersection of cutting plane with triangulated surface) and its length (L) are determined and then divided into L/ R_p vertex points, separated equally. Interior points, selected at random but which are not part of an existing patch, are grown next. Patch growth is similar to the preceding step but with radius R_p. After a patch is added, its circumference is checked to make sure the patch is homeomorphic to a disk. If not, new patches (of lesser radius) are added inside the offending patch until all homeomorphisms are disk-like. Surface curvature (_$_1$) is used to locally reduce P_r, if needed, thus reducing frequency of such breakdowns, and providing automatic surface refinement. The algorithm completes when the surface is completely covered with patches and 1) every vertex in triangulation shares three or less patches, 2) patches share at most one edge with every other patch. The result of this step for the pulmonary trunk is shown in figure 2.

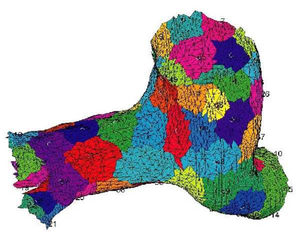

Figure 2

Step 3: Generate Edges

The rationale for the above careful construction of the Voronoi Diagram is that the dual to the Voronoi diagram is the Delaunay triangulation. That is, by connecting each of the vertices in the previous step across shared edges, we have a Delaunay triangulation. The edges we have chosen are the geodesics between patch vertices, and they can be calculated by either streamline integration of the Eikonal Equation solution, or a simple initial vertex descent along Eikonal solution followed by segment length minimization. Streamline integration, while slower, produces smoother curves.

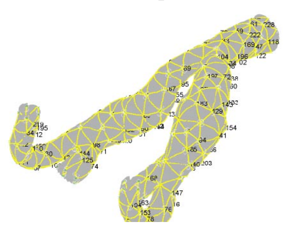

Figure 3

Once the geodesic paths are known, they are fit to cubic Hermite line segments using an adaptive least squares method applied to the nonlinear parametric curve. Fitting proceeds by fixing the endpoints, and solving for the tangent vectors that minimize the distance between N equally spaced points on the geodesic and the cubic Hermite curve. Each fit proceeds iteratively, with a least squares fit followed by a Gauss-Newton minimization of parameter values for the N points. The result of this step is shown in figure 3.

Step 4: Fit Tensor Product Surface Patches

At this point, the triangles could be surface fit; however, many commercial mesh algorithms require square input patches and so triangles are paired edgewise to generate squares (except for a few "leftover" triangles treated as degenerate squares). The cubic edges from the previous step are used directly as data for a bicubic hermite tensor product patch. Twist vectors are set to zero, thus producing a Ferguson patch. This patch is easily converted via matrix multiplications to Bezier or B-spline forms if desired. The completed cubic Hermite surface is shown in Figure 4. This surface is easily input into a mesh generator: after the "open" ends are capped with planar triangular patches, the surface is unioned and then meshed.

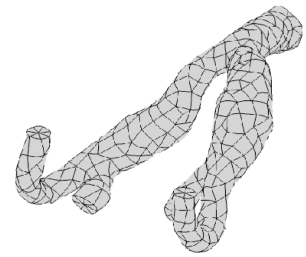

Figure 4

One Dimensional Extensions

By using the Eikonal equation on the distal vasculature, a 1D representation of the distal tree can be obtained by tracking the topology of the "rings" generated by the Eikonal equation. If these rings split into two, then a vessel has branched. Initially, the center points of the rings are used as nodes in a 1D path. At each node, the minimal circumferential path is calculated, and from this the area is calculated and the node recentered. A 1D extension structure (corresponding to one of the trimmed branches of figure 2) is shown in Figure 5.

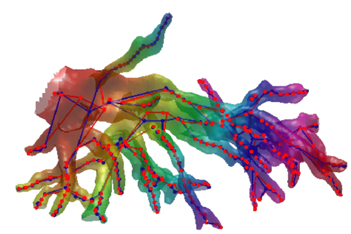

Figure 5

DISCUSSION

Further work will be directed toward application of the method to complex CT and MR 3D data sets including: the Circle of Willis, complete pulmonary tree, and congenital vascular disease cases.

ACKNOWLEDGMENTS

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