NUMERICAL INVESTIGATION OF CIRCADIAN VARIATION IN STATURE

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INTRODUCTION

Low back pain is one of the most prevalent and costly work related injuries in the United States. Although epidemiological studies have suggested possible causes, the actual mechanisms by which the lumbar spine is injured during repeated load cycles resulting in low back pain, remains unknown. It is the unique interaction between the solid and fluid components that provide the disc the strength and flexibility required to bear cyclic loading of the lumbar spine. However, the response of the disc to the cyclic loading conditions that occur during repetitive lifting is difficult to measure in vivo and in vitro and has not been investigated using any model.

Therefore, the purpose of this study was to construct a numerical model that predicts the loading and unloading behavior of a lumbar motion segment including the poroelastic behavior of the disc, the effect of the change in proteoglycan content in the disc and the effect of strain dependent permeability with respect to applied loads.

MATERIALS AND METHODS

A three-dimensional finite element model of the L4-L5 motion segment, which was validated for static loading conditions [1], was modified to include the poroelastic components. Initial permeability and porosity values for the nucleus, annulus, endplate, cancellous bone and cortical bone were taken from the literature [2]. The drained elastic modulus and Poisson's ratios for all the disc components were also taken from the literature [2].

The inclusion of poroelastic components in the finite element model introduced the effect of fluid within the intervertebral disc. The effect of the change in the concentration of proteoglycans contained within the nucleus was modeled by incorporating a pressure, in this case referred to as the swelling pressure (p_{swell}), which is dependent on the fixed charge density. Using equations defined by Broberg and by Urban et al. [3, 4], the initial fixed charge density can be calculated for each increment in the load, knowing the initial volume of the disc, based on geometric dimensions from the finite element model, and the water content of the disc, in this case assumed to be 70% [4]. The swelling pressure is then calculated using an equation proposed by Broberg [3]:

$$p_{swelli} = Pf_i \frac{f_i^2 + 1}{a\, f_i^2 + 1} \quad \ldots\ldots\ldots\ldots \text{(1)}$$

where f_i is the fixed charge density at time t_i and P and a are constants.

Similarly, the effect of the change in permeability resulting from the axial strain in the tissue was included through an internal pressure acting on the disc, which accounts for the dilatation of the pores as the tissue is compressed. From the results of the finite element model for each increment in the load, the corresponding change in axial disc height can be calculated and subsequently the axial strain (e) in the disc tissues can be calculated as:

$$e_i = \frac{h_o - h_i}{h_o} \quad \ldots\ldots\ldots \text{(2)}$$

where h_o and h_i are the initial disc height and the disc height at time t_i respectively. This axial strain can then be used to calculate the change in the tissue porosity:

$$f_i = \frac{e_i + f_o}{1 + f_o} \quad \ldots\ldots\ldots\ldots \text{(3)}$$

where f_o is the initial porosity for the tissue. The pressure that is exerted by the fluid onto the solid matrix as the pores dilate referred to here as p_{strain} is a component of the Cauchy stress tensor for the solid phase:

$$s^s = -f\, p_{strain} + s^e \quad \ldots\ldots\ldots\ldots\ldots \text{(4)}$$

where σ^e is the elastic portion of the solid stress. Assuming that the deformation in the transverse plane is approximately zero when compared to the deformation in the axial direction, the constitutive relation for the axial component of Cauchy stress and axial stretch can be expressed as [6]:

$$s^e = \frac{1}{2} H_A \frac{(e_i+1)^2 - 1}{(e_i+1)^{2b+1}} \exp\left[b\left((e_i+1)^2 - 1 \right) \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

Where H_A is the zero-strain aggregate modulus and b is the non-linear stiffening coefficient. Using equations (3) and (5) in equation (4) the pressure (p_{strain}) can be written as a function of strain [6]:

$$p_{strain_i} = \frac{(Ee_i) - \left(\frac{1}{2} H_A \frac{(e_i+1)^2 - 1}{(e_i+1)^{2b+1}} \exp\left[b\left((e_i+1)^2 - 1 \right) \right] \right)}{-f_i} \quad \ldots\ldots\ldots\ldots\ (6)$$

Using the finite element model, the lumbar motion segment was loaded to observe the circadian variation in disc height. The disc was loaded equivalent to normal daily activities without participating in physical work or exercise for approximately 16 hours and sleep was simulated for approximately 8 hours. Normal daily activity was modeled by applying a compressive load of 850 N [2] that was reduced to 440 N to simulate the load during sleeping. The change in disc height predicted by the finite element model was compared with the measured height changes in normal subjects as reported by Tyrrell et al. [5].

RESULTS

The finite element model results showed a gradual increase in disc height loss over the 16-hour period of activity with a maximum height loss of 1.38 mm, a 13% loss of total disc height (Figure 1). This distribution compared well with the experimental results, in which a maximum disc height loss of 1.3 mm was observed [5]. The finite element results predicted that 42% of the total diurnal disc height loss occurred during the first hour of loading as compared to 43% observed in vivo by Tyrrell et al. [5]. Upon releasing the load, a sudden recovery of height was observed both with the finite element model results as well as in in-vivo results. During the first 4 hours of recovery, the finite element model predicted 80% the total disc height loss was recovered as compared to approximately 70% observed in vivo [5]. As the recovery time increased, the rate at which the height was recovered decreased and began to plateau. The finite element results showed a complete recovery of disc height at the end of the 8 hours of recovery, which was in agreement with the in vivo data reported by Tyrrell et al. [5]. Throughout the 24-hour period, the variation in disc height observed with the finite element model fell within the standard deviation of the reported in vivo results.

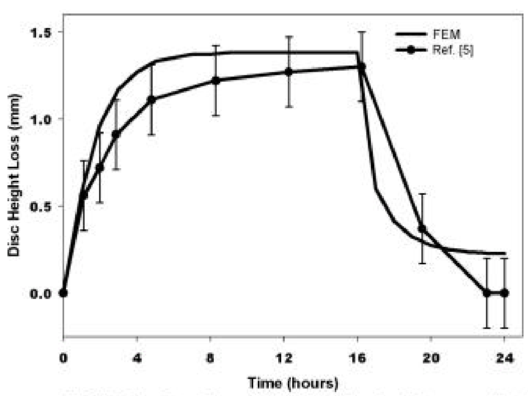

FIGURE 1. Circadian variation in disc height as predicted by FEM compared to in-vivo results [5].

As the disc is compressed, fluid is squeezed out of the disc into the surrounding tissues. This loss of fluid is reflected in the increase in fixed charge density (f), which results in an increase in the swelling pressure (p_{swell}) as described in equation (1) (Figure 2).

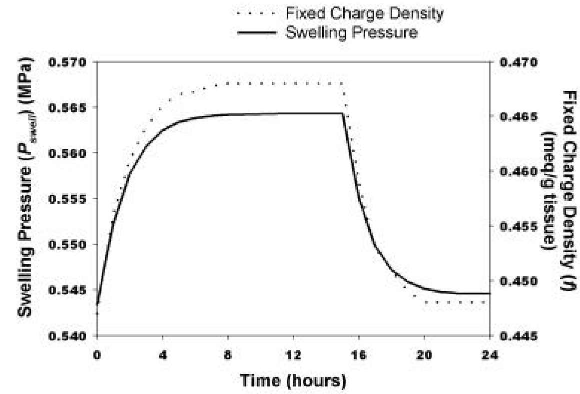

FIGURE 2. Circadian variation in swelling pressure (P_{swell}) and fixed charge density (f).

Similarly, as the disc tissue is compressed, the axial strain increases and resulting in the dilatation of the pores in the solid matrix. This results in a decrease in porosity and subsequently inhibits the flow of fluid out of the disc. This restriction in flow results in an increase in the pressure (p_{strain}) (Figure 3).

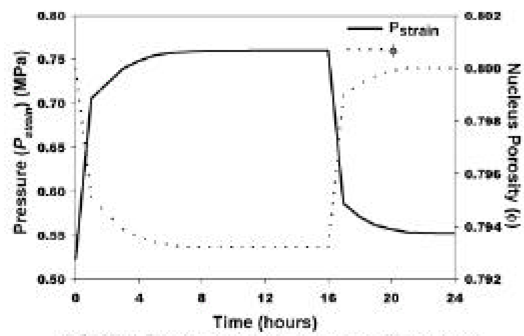

FIGURE 3. Cicadian variation in the pressure (P_{strain}) and nucleus porosity (ϕ).

DISCUSSION

A numerical model has been constructed that predicts the loading and unloading behavior of a lumbar motion segment over a 24-hour period. The finite element model predicted that nearly 50% of the total diurnal disc height loss occurred during the first hour of loading. It also predicted that most of the disc height recovery occurred during the first 4 hours of recovery similar to those observed in healthy subjects [5].

This numerical model can now be used to study the response of the disc to various cyclic loading conditions that occur during repetitive lifting.

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ACKNOWLEDGMENTS

NIH: AR48152-02, The first author wishes to acknowledge financial support from Rush-Presbyterian-St. Luke's Medical Center.