A DAMAGING-VISCOELASTIC-VISCOPLASTIC CONSTITUTIVE MODEL FOR HUMAN VERTEBRAL TRABECULAR BONE APPLIED TO EXPERIMENTAL SPECIMEN RESPONSE

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INTRODUCTION

Continuum damage mechanics (CDM) models, particularly when incorporated into finite element models, offer the potential for characterizing the damage accumulation process and the risk of fracture in real skeletal structures. For example, Zysset and Curnier applied a quite general inviscid, isotropic damage model to cancellous bone around the central assumption that plastic flow and damage accumulation are intrinsically related [1]. With evidence from our own studies [2] and others that time-dependence [3] and damage anisotropy [4] are significant in trabecular bone and a lack of specific experimental basis for assuming an intrinsic link between plastic flow and damage, a unified constitutive model was developed to describe the elastic and inelastic (viscous, plastic, and damage) behavior of human vertebral trabecular bone. This constitutive model was implemented in finite element simulations using a commercial finite element analysis (FEA) solver (ABAQUS, ABAQUS, Inc.), along with parameters determined from experimental data and the literature, to predict the mechanical response of trabecular bone.

METHODS

An orthotropic constitutive model for human vertebral trabecular bone [2] incorporating viscoelastic and damage effects was expanded using phenomenological associated material models, specifically through the development of a plasticity model to describe permanent stresses and a damage mechanics model to describe the reduction in apparent stiffness, allowing independent plastic and damage responses.

An orthotropic plasticity model was developed in stress-space to include a full multiaxial (Tsai-Wu) yield criterion and isotropic plastic hardening, both related by orthotropic stiffness tensor to trabecular density and architecture. By combining strength-density and modulus-density regression equations [5], orthotropic tensile and compressive yield strengths were estimated in terms of orthotropic moduli on the basis of the strong correlation between axial yield strength with axial modulus for trabecular bone, regardless of specimen orientation [6] and shear strengths were determined from shear moduli assuming transversely-isotropic shear behavior [7]. Tsai-Wu coefficients were determined using normal and shear strength data [8] and enforcing positive-definiteness of the tensor of Tsai-Wu coefficients. Isotropic hardening was assumed and a hardening modulus was estimated as the slope of the ideal linear, uniaxial stress-strain response between yield and ultimate points, where stress values were related to the primary initial elastic modulus and strains were taken as mean values [5].

A damage model was developed previously [2] using strain energy-equivalence to incorporate the effects of orthotropic damage accumulation. This damage-space model was extended using a damage energy release rate vector to express energy dissipation due to damage propagation, and to define a quadratic damage potential surface, where the damage characteristic tensor describes the anisotropic nature of damage growth and was defined as a symmetric tensor function of the ratios of damage accumulation in the three orthogonal axes [9, 10], and an energy threshold used to predict damage propagation was determined at the onset of experimental damage, approximated for practical purposes by a 0.5% reduction in modulus. To avoid solution divergence associated with a static damage potential, the damage surface was allowed to uniformly expand as a function of damage strengthening, directly comparable to plastic hardening. Damage strengthening modulus was determined by regressing the experimental damage strengthening stress against the overall accumulated damage history [10].

Assuming small strains, equations describing the viscous, plastic, and damage behavior were reformulated and combined assuming additive decomposition of the total strain tensor [11]. Given a total strain increment, a unified algorithm was used to 1) account for viscous effects, 2) iteratively solve for plasticity and damage solutions within stress and damage space, respectively, 3) calculate apparent stress, and 4) calculate a unified tangent moduli matrix representing a linear Jacobian transformation between apparent stress and total strain.

This unified constitutive model and solution algorithm were implemented as a user-defined material in ABAQUS, along with constitutive parameters determined for a typical cylindrical trabecular bone specimen with superior-inferior (SI) orientation and used to define a base model with geometry composed of 20-node hexahedral

FEA elements. This base model was used to simulate an experimental uniaxial strain-controlled protocol composed of a trapezoidal pulse with a peak strain of 0.8% and a 60 sec. hold period, a 180 sec. recovery period, and a monotonic (reloading) ramp to 1.2% strain, with loading and unloading rates of 0.5%/sec.

Base parameters were individually varied to investigate parameter sensitivity: the time constant was varied by ±50%, yield strengths were varied by ±5% (altering Tsai-Wu coefficients and hardening modulus), the damage threshold was set to zero, and the damage characteristic tensor was changed to reflect damage orthotropy ratios predicted by Niebur, *et al.* [4] and isotropic damage growth [1].

RESULTS

The base FEA model was able to predict the first loading ramp response quite well and captured the basic features of the reloading ramp, including the stress plateau. However, the FEA response overshot the experimental response in the middle portion of the reloading ramp and underestimated the permanent strain and hysteresis from the trapezoidal pulse (Figure 1).

Uniaxial stress predictions were minimally affected by variation in time constant or damage characteristic tensor. Also, variation of yield strength components produced small variation in stress response, occurring in the stress plateau following apparent yield. Allowing immediate damage propagation (*i.e.* zero damage threshold) generally improved the reloading response, although the stress plateau was underestimated (Figure 2).

Percent reduction for initial tangent moduli [2] and perfect-damage moduli [12] were determined for base FEA and experimental responses. Although FEA tangent moduli were lower than corresponding experimental values, reductions in tangent moduli were reasonably close (8.06% compared to 11.72%). Perfect-damage moduli for FEA and experimental responses were strikingly similar, and reductions in perfect-damage moduli were 31.95% and 33.33%. Axial residual strain was calculated as a function of tangent modulus and perfect-damage modulus [12] for base FEA and experimental responses. These strains were 0.098% and 0.140% following the trapezoidal pulse, and 0.217% and 0.262% following reloading. Parametric variation did not significantly improve these comparisons.

DISCUSSION

A phenomenological constitutive model has been implemented in conjunction with a computational solution scheme to describe the elastic and inelastic response of human vertebral trabecular bone to applied loading. The ability to predict damage measures, such as evolving and accumulated modulus degradation, was achieved despite the complicated nature of the material model and the limited amount of data for multiaxial material characterization. The FEA simulation seems to bear out the basic assumptions of the model and demonstrates that it is possible to obtain estimates of all model parameters based on experimental data and the literature.

The mechanical behavior of trabecular bone has been investigated with microstructural models using FEA methods [4, 13]. While there are obvious advantages to models incorporating trabecular structure, they quickly become very complex and computationally intensive, even with simple models for tissue behavior [13]. When complex material behavior is considered, continuum-based models offer the capability to examine realistic structures with reasonable computational costs. The ability to model the basic features and reasonably predict the highly nonlinear behavior of low-density heterogeneous vertebral trabecular bone allows us to move somewhat closer to the goal of predicting the *in vivo* response or risk of fracture in a clinical setting.

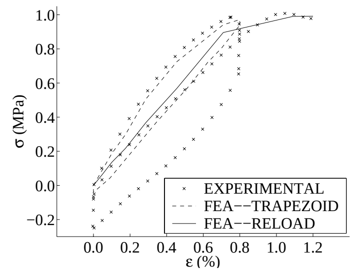

Figure 1. Experimental vs. base model SI response.

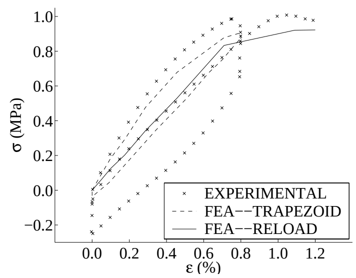

Figure 2. Experimental vs. threshold = 0 model SI response.

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