LINEARIZED CONTROL OF FORWARD DYNAMIC BIPEDAL WALKING

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INTRODUCTION

Active joint torques are the primary stabilizing factor in dynamic motion. However the rate, timing, phasic behavior and amplitude of the joint torques necessary to achieve stable performance are difficult to establish. Mochon [3] introduced a Ballistic Gait model, demonstrating that the swing phase of walking can be successfully achieved using an unpowered multilink pendulum. To quantify dynamic walking stability, McGeer [2] implemented a two-segment pendulum-walker and observed that limit-cycle stability was intrinsic to the passive mechanics of multi-stride downhill walking. To assure continuous walking, periodic energy input is necessary to maintain stable locomotion. In McGeer's study this input energy was achieved by walking downhill thereby limiting the applicability of the analyses. Step length and speed, for a given mechanical configuration (hip and leg masses and leg length), were a function of the slope, i.e. added energy rate. However, to achieve stable walking dynamics on level ground active input energy must be applied to the forward-dynamic simulations.

Virtual slope control of active bipedal walking introduced the concept that walking can be controlled on level ground in a manner that simulates downhill passive walking. Note that no active joint torques are required for downhill walking stability but gravitational joint torques are quantifiable. Recognizing that passive downhill walking is dynamically stable, then virtual slope control must achieve stable walking dynamics in active bipedal systems. This was validated using a two-segment pendulum walker, and provided additional benefits wherein the step-length and speed were explicitly controlled by selecting the appropriate virtual slope [4].

The nonlinear controller resulted in a large bias torque τ_0 with small variations based on the joint angles, θ, throughout each stride. Hence, we hypothesized that a simple linear controller can be used to apply these active joint torques and maintain stable dynamics.

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 - \mathbf{G}\boldsymbol{\theta} \tag{1}$$

where $\boldsymbol{\tau}$ is the vector of joint torques applied at the ankle and hip of the pendulum-walker. The bias torque, $\boldsymbol{\tau}_0$ and linear feedback gain, $\mathbf{G}$, are functions of virtual ground slope and can be used to explicitly control step-length and walking speed. The goals of this study were to implement the linear, virtual-slope, control system for joint torques then demonstrate controllability of step-length and stability by means of forward-dynamic simulation of the pendulum-walker.

METHODOLOGY

A forward dynamic model was successfully developed and used to quantify stability and define the initial conditions associated with the region of stability [1]. The simulation represents a 2-dimensional knee-less walker including two legs of length L and mass M_L, joined by a revolute joint at the point mass of the head-arms-trunk, M_H (Figure 1). Anthropomorphic data including segment lengths, mass, and mass distribution were based upon physical attributes of an average adult male. During walking only one foot is in contact with the ground at any time. Ground clearance of the swing-leg is ignored because simple mechanisms such as prismatic joints are readily established that do not influence walker dynamics. Movement dynamics were determined from two, coupled, nonlinear, second-order differential equations of motion describing a double-pendulum with a pivot at the stance foot. The collision at foot-strike was represented as a plastic collision and the transition stage at foot-strike was assumed instantaneous, i.e. no double-support period. Angular velocities before and after foot strike were related by conservation of angular momentum as described by Goswami [1].

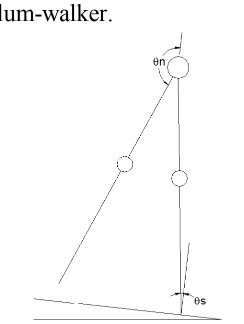

Figure 1. Compass Model

Gravitational joint torques, $\boldsymbol{\tau} = [\tau_{\text{Ankle}}, \tau_{\text{Hip}}]^T$, of the passive pendulum-walker can be expressed in terms of the leg angles, θ_S and θ_N, and ground slope γ, the gravitational constant g, and a mass matrix.

$$\begin{bmatrix} \tau_{Ankle} \\ \tau_{Hip} \end{bmatrix} = -g \begin{bmatrix} (M_H + 2M_L)L - M_L u_{CM} & 0 \\ 0 & M_L u_{CM} \end{bmatrix} \begin{bmatrix} \sin(\theta_S - \gamma_V) \\ \sin(\theta_N - \gamma_V) \end{bmatrix} \tag{2}$$

Clearly, on level ground surfaces active joint torques, $\boldsymbol{\tau}$, can be applied that simulate walking down a virtual slope, γ_V. Natural dynamic step-length and speed of a passive walker are functions of ground slope [2], and can be approximated as (solid line in Figure 2)

$$\gamma_V = \frac{\theta_0^3}{\theta_0^2 - \beta} \quad (3)$$

where stride-length is dictated by leg angle at foot strike, θ_0, and β is a function of radius of gyration of the legs. We have previously demonstrated that step-length and walking speed can be explicitly controlled by appropriately specifying virtual slope, γ_V for a desired foot strike angle θ_0, using a similar nonlinear active controller on downhill, level and uphill grades [4]. For improved simplicity the active virtual torque control (eqn 2) can be linearized in leg angle $\boldsymbol{\theta} = [\theta_S, \theta_N]$ to the form of control equation (eqn 1) with coefficients $\boldsymbol{\tau}_0$ and **G** that are functions of desired step-length, i.e. virtual slope,

$$\mathbf{G} = \tfrac{1}{2}\, g\,\mathbf{M}(\cos(\gamma_V)-1), \quad \boldsymbol{\tau}_0 = g\,\mathbf{M}\cos(\gamma_V) \quad (4)$$

where **M** is the mass matrix in equation 2. Substituting equation 3 into the control parameters the linear system can be designed to walk at pre-specified step-lengths and speeds.

Analysis of limit cycle stability for the forward-dynamic model was performed numerically as described in the literature [1,2]. Briefly, initial state of the system at step q_k following from the initial state at step $q_k = f\,(q_{k-1})$. If the state is perturbed Δq then a Taylor series representation of the response is described as

$$f(q_{k-1} + \Delta q) = q_k + \Delta q\, \nabla f \quad (5)$$

where ∇f is the gradient of the stride function. The perturbed trajectory converges toward a stable steady state behavior if the Eiganvalues of ∇f are less than one. By introducing a perturbation Δq to each of the state variables at q_{k-1} and observing the response q_k a numeric representation of ∇f is achieved to quantify stability. Forward-dynamic simulation of the actively powered pendulum-walker was implemented in MATLAB (Natick, MA) and tested for its ability to control step-length and stability using the linear controller.

RESULTS

A linear feedback controller for active ankle and hip torques based on passive downhill walking was designed to achieve the natural walking behavior of a bipedal pendulum-walker. The behavior of the walker when controlled through active joint torques with virtual slope γ_V, was nearly indistinguishable from the natural dynamics of a passive pendulum-walker at an equivalent downhill ground slope. The equilibrium stride length of the linear controller varied less than 0.6% of the non-linear stride length, and the angular trajectories varied less than 0.15% and 0.67% for the ankle and hip respectively. This natural behavior of the actively controlled walker was independent of the actual ground slope; even uphill walking was readily achieved.

Stable gait dynamics were empirically recorded from the active behavior of the linear, virtual-slope pendulum-walker. Eigenvalues were relatively insensitive to virtual slope or ground slope for small state perturbations. However, the nonlinear stability of the governing equations and controller was evident by virtue of the fact that the eigenvalues increased with state perturbation amplitude. Moreover, the stability was more sensitive to virtual slope or ground slope when larger perturbations were investigated. Clearly the stability is limited in extent, observable from eigenvalues that exceed one. Nonetheless, within reasonable limits the results demonstrate the linear virtual slope active controller achieved dynamically stable gait patterns in the pendulum-walker. The maximum perturbation magnitude for which the walker remained stable varied less than 0.79% between the non-linear and linear control.

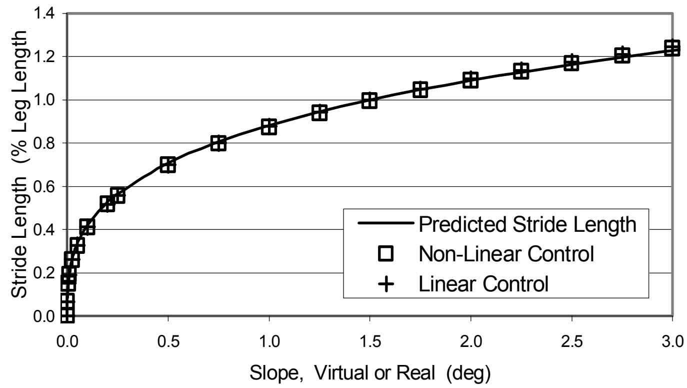

Figure 2: Comparison of Resulting Step-Lengths

CONCLUSIONS

Previously the virtual slope method has been implemented to control a simple bipedal walker in a stable manner while still allowing for the natural motion based on the natural dynamics of an inverted double pendulum. The purpose of this research was to explore the extent to which this virtual control can be simplified. Linearization is the first obvious step in the simplification process. This results in a controller based on a large bias torque component, $\boldsymbol{\tau}_0$, and a smaller feedback gain, **G**. Both control components are based solely on the desired step-length of the model. Linearization errors become significant at 10°-15° while in this model peak leg angles were observed at less than $\pm$ 19°. However, the feedback gain, **G**, is less than 5% of the bias torque, $\boldsymbol{\tau}_0$, for small virtual slopes (equation 4). Thus, the linearization errors do not adversely affect performance of movement trajectories or stability.

Due to the similarities of the model behavior using both the non-linear and linearized torque control, the linearized control is a reasonable simplification and replacement for the non-linear control. This is significant in that it results in less computationally intensive design. Benefits can be realized in robotics where computation time is a significant constraint. Moreover, modeling of biological walkers were the idea of a central pattern generator relies on simple basic controls to generate simple motion, and relies on higher level control feedback for fine motor control. It is this second field which we will likely implement this research in the future.

Due to the small contribution of the feedback gain future studies will investigate the use of a constant joint torque. Preliminary models have demonstrated stable gait patterns developed applying constant torque as the only input. It is believed this is possible due to a naturally occurring stride length feedback [2,4].

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