VELOCITY BOUNDARY CONDITIONS FOR SUBJECT-SPECIFIC ARTERIAL FLOW SIMULATIONS

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ABSTRACT

Computational fluid dynamics (CFD) coupled with *in vivo* imaging has allowed subject-specific and patient-specific flow simulation, leading to the possibility of clinical applications. Where arterial velocity images as well as anatomical images are acquired, the subject-specific velocity data may be used as boundary conditions. When such data are not available, careful consideration should be given to their specification. The application to the human descending thoracic aorta is discussed.

INTRODUCTION

With the advance of non-invasive medical imaging, particularly magnetic resonance (MR), enabling the acquisition *in vivo* of the anatomy of arteries and their 3D reconstruction for analysis, has come the possibility of application in CFD flow simulations. Such flow simulations may be applied, for example, to investigations of basic physiology (Wood *et al.*, 2001; Saber *et al.*, 2001), in evaluating the mechanical determinants of regulation and disease (Zhao *et al.*, 2000), and in assisting patient diagnosis and treatment (Taylor *et al.*, 1999). A requirement for the solution of differential equations, like the Navier-Stokes equations for fluid flow, is the specification of case-specific boundary conditions. In the present application, appropriate boundary conditions usually include the time-varying inlet velocity profiles, which will incorporate the effects of conditions proximal to the computational domain. These may be available from MR velocity imaging, but not always. In these cases, an alternative must be found. The use of profiles obtained via the axisymmetrical long tube approximation at the appropriate Womersley parameter is generally better than the Hagen-Poiseuille profile, but if the proximal anatomy includes curvature and branches, the true profiles may differ significantly from either.

ILLUSTRATION

The example discussed here is the human descending thoracic aorta, previously reported by Wood *et al.* (2001). Briefly, MR anatomy scans of a 110 mm length of the vessel, starting just distal to the left subclavian branch and ending in mid thorax, was acquired, together with time-resolved velocity profile scans at both inlet and outlet planes. A computational mesh was developed for the flow domain, which was assumed rigid, and the flow was simulated *via* the finite volume CFD code STAR-CD, using the measured flow for inlet boundary conditions and a uniform pressure condition at the exit.

An important feature of the inlet conditions was that a complex flow pattern, with large secondary components, developed in late systole as the flow retarded, originating from the aortic arch, owing both to the curvature and the flow out of the bend to the brachio-cephalic branches. As observed in several subjects by Kilner *et al.* (1993) a high-speed jet developed at the inside of the bend as the flow exited from the arch. During late systole the flow separated and the flow pattern was influenced by this throughout the domain. Details of the simulated developing flow pattern is shown and discussed.

DISCUSSION

The importance of the above result is that frequently there are reasons why velocity profile data are not available for use as boundary conditions for flow simulations. For example, a patient may be unable to tolerate the scanner for sufficient time, or if ultrasound scanning is used, only the centerline velocity might be available. In another case involving flow into the left ventricle (Saber *et al*., 2003), there was a change in pulse rate between the anatomical and velocity scans. Moreover, the inlet velocity distribution was distorted by the flow from the left inferior pulmonary vein, which entered the left atrium parallel to the plane of the mitral valve ring, and just above it, causing a separation at the mitral valve and raised trans-valvular velocity levels.

When measured velocity data are unavailable for flow in an artery, the use of a pulsatile flow solution at the appropriate Womersley parameter for a straight artery might be considered the best that can be done to provide them. However, if the arterial geometry proximal to the computational domain possesses a major feature such as a bend or a bifurcation, a better approximation would be given by modeling the feature in order to provide the required conditions. Even so, the approximation might give only an indication of the true proximal effects on the flow.

Sometimes, an analytical expression is wanted to provide an approximate velocity boundary condition, and frequently recourse is made to the Hagen-Poiseuille profile. Whilst this is valid for the mean flow, or in time-varying form as an approximation for low values of the Womersley parameter, better approximations are available for higher values of the parameter (Sexl, 1930; Uchida, 1956). As an example, for an axial pressure gradient of

$$- \partial p / \partial x = \rho(P_0 + P\cos\omega t) \quad (1)$$

where P is the pressure amplitude, the subscript 0 referring to the time-mean amplitude, the approximation for velocity near the wall is given by

$$u/U = F_0 + \frac{P}{P_0}\frac{8}{\alpha^2} \times \left[\sin\omega t - \sqrt{a/r}.\exp(-\beta)\sin\{\omega t - \beta\} \right] \quad (2)$$

where

$$\beta = \frac{\alpha}{\sqrt{2}}(1 - \frac{r}{a})$$

and near the center by

$$u/U = F_0 + \frac{P}{P_0}\frac{8}{\alpha^2}\cos(\omega t - \frac{\pi}{2}) \quad (3)$$

where U_0 is the total mean velocity, u is the instantaneous velocity at radius r (origin at the center), a is the tube radius, α is the Womersley parameter and F_0 is the Poiseuille distribution. Some blending may be required between the expressions to provide continuous profiles. These approximations were used by Wood (1999) to calculate velocity distributions for $\alpha = 5$ and 10. In order to simulate an arterial waveform, additional harmonics may be added (Womersley, 1955).

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REFERENCES

Kilner, P. J. *et al.*, 1993, “Helical and retrograde secondary flow patterns in the aortic arch studied by three-directional magnetic resonance velocity mapping,” *Circulation*, Vol. 88, pp. 2235-2247.

Saber, N. R. *et al.*, 2001, “Computational flow modeling of the left ventricle based on *in vivo* MRI data – initial experience,” *Annals Biomed. Eng.,* Vol. 29, pp. 275-283.

Saber, N. R. *et al.*, 2003, “Progress towards patient-specific computational flow modeling of the left heart via combination of MRI with CFD.” *Annals Biomed. Eng.,* in press.

Sexl, T., 1930, “Über den von E. G. Richardson entdeckten ‘Annulareffekt’,” *Z. Phys*., Vol. 61, pp. 349-362.

Taylor, C. A. *et al.*, “Predictive medicine: computational techniques in therapeutic decision-making,” 1999, *Comput. Aided Surgery*, Vol. 4, pp. 231-247.

Uchida, S., 1956, “The pulsating viscous flow superposed on the steady laminar motion of incompressible flow in a circular pipe,” *ZAMP*, Vol. VII, pp. 403-422.

Womersley, J.R., 1955, “Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known,” *J. Physiol*., Vol. 127, pp. 553-563.

Wood, N. B., 1999, “Aspects of fluid dynamics applied to the larger arteries,” *J. Theor. Biol*., Vol. 199, pp. 137-161.

Wood, N. B. *et al*., 2001, “Combined MR imaging and CFD simulation of flow in the human descending aorta,” *J. Magn. Reson. Imaging,,* Vol. 13, pp. 699-713.

Zhao, S.Z. *et al*., 2000, “Blood flow and vessel mechanics in a physiologically realistic model of a human carotid arterial bifurcation,” *J. Biomechanics*, Vol. 33, pp. 975-984.