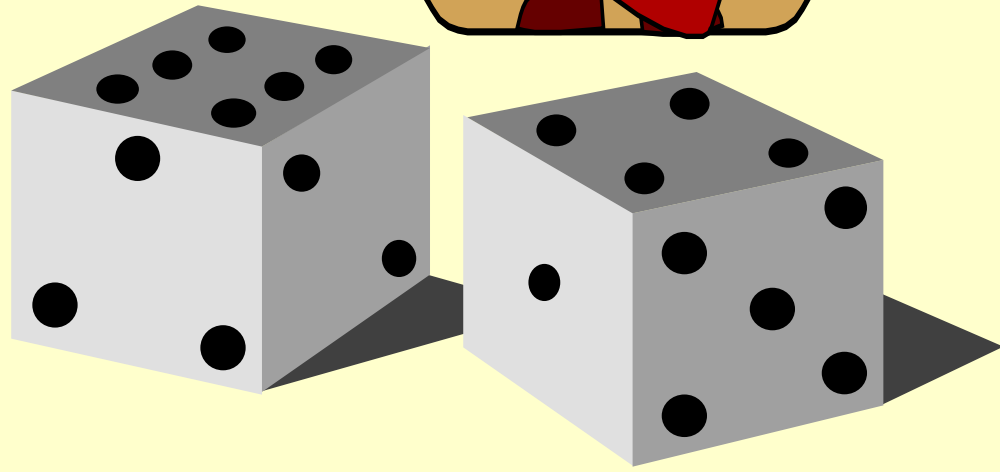
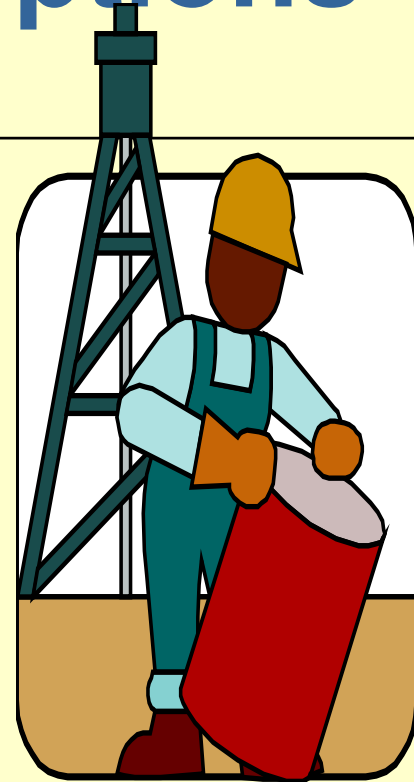


Pricing Energy Options

Optionality is Reality



Overview of Lecture

- Aim of talk: understand how option pricing is used to price energy options
- What you need to know
 - ◆ Why use real options approach
 - ◆ How real option valuation works
 - ◆ What are the special problems linked to applying real options in the energy sector

The Problem: Large Scale Capital Investment Projects

- Large Scale Capital Investments
 - ◆ Long-time horizons
 - ◆ Significant risks
 - ☞ Project specific
 - ☞ Macroeconomic
 - ◆ Irreversibility

Large Scale Capital Budgeting Project

- ◆ Given the complexity of the problems and the stakes involved, it is worthwhile to put serious effort in to getting the valuation right.
- ◆ Correct valuation of any project is obtained by

Discounting expected cash flows, at the project specific cost of capital.

The project specific capital cost depends on the fine details of project implementation

Example 1: Classic Energy Real Options Problems

- Value a generation asset: a peaker power plant
 - ◆ High marginal cost (typically gas turbine) plant that can be ramped up and down at low cost.
 - ◆ Underlying assets: price of input fuel (F) and price of electricity (E)
 - ◆ Relation between assets and project cash flows at date T

$$\text{Max}[E_T - h * F_T - c, 0]$$

Ex 2: Classic Energy Real Options Problems

- Value transmission assets
 - ◆ Build transmission asset that will allow electricity (or gas) to be sold in a new market.
 - ◆ Fixed cost of transmission asset, per period, C .
 - ◆ Benefit, choice between markets
 - ◆ Per-period benefit
$$(\text{Max}[E_T^1, E_T^2] - E_T^1) - C$$

Ex 3: Classic Energy Real Options Problems

- Value dual-fuel power plant
 - ◆ Plant sells output at fixe price e
 - ◆ Cost of fuel input j : C_j
 - ◆ Heat exchange rate for fuel j : h_j
 - ◆ Per-period payoff

$$e - \text{Min}[h_1 C_1 , h_2 C_2]$$

What's different about the RO approach

- Classical discounted cash flow valuation techniques:
 - ◆ Evaluate projects by taking the expected or “most likely” values of parameters.
 - ◆ And discounting at the industry or firm cost of capital. For example in one period setting

$$\text{Value} = \frac{\text{ProjectPayoff}(E[\text{ProjectParameters}])}{1 + k_{\text{Firm}}}$$

What's different about RO approach

- Traditional Monte Carlo Analysis:
 - ◆ Simulates cash flows for different possible values of project parameters
 - ◆ Takes expectations (averages)
 - ◆ Discounts at the industry or firm cost of capital

$$\text{Value} = \frac{E[\text{ProjectPayoff}(\text{ProjectParameters})]}{1 + k_{\text{Firm}}}$$

RO approach

- Real Options Analysis
- Discounts expected project cash flows (like traditional MonteCarlo)
- At a discount rate that depends on the *precise* way the project is managed

$$\text{Value} = \frac{E[\text{ProjectPayoff}(\text{ProjectParameters})]}{1 + k_{\text{Project}}}$$

But how and you do that?

- Basic Theorem of financial economics is the no-arbitrage theorem

$$\frac{E^*[\text{ProjectPayoffs}]}{1 - k_{\text{RiskFree}}} = \frac{E[\text{ProjectPayoff}(\text{ProjectParameters})]}{1 + k_{\text{Project}}}$$

E^* : expectations under the risk-adjusted probability measure.

But what are risk adjusted probabilities?

Risk-adjusted probabilities

AKA:

pseudo-probabilities,

pricing probabilities

martingale measures

implied probabilities

Risk-neutral probabilities

Risk adjusted probabilities are probabilities (that is, positive and sum up to 1) that reflect the market's valuation of cash flows in different states of the world.

How do we get risk adjusted probabilities?

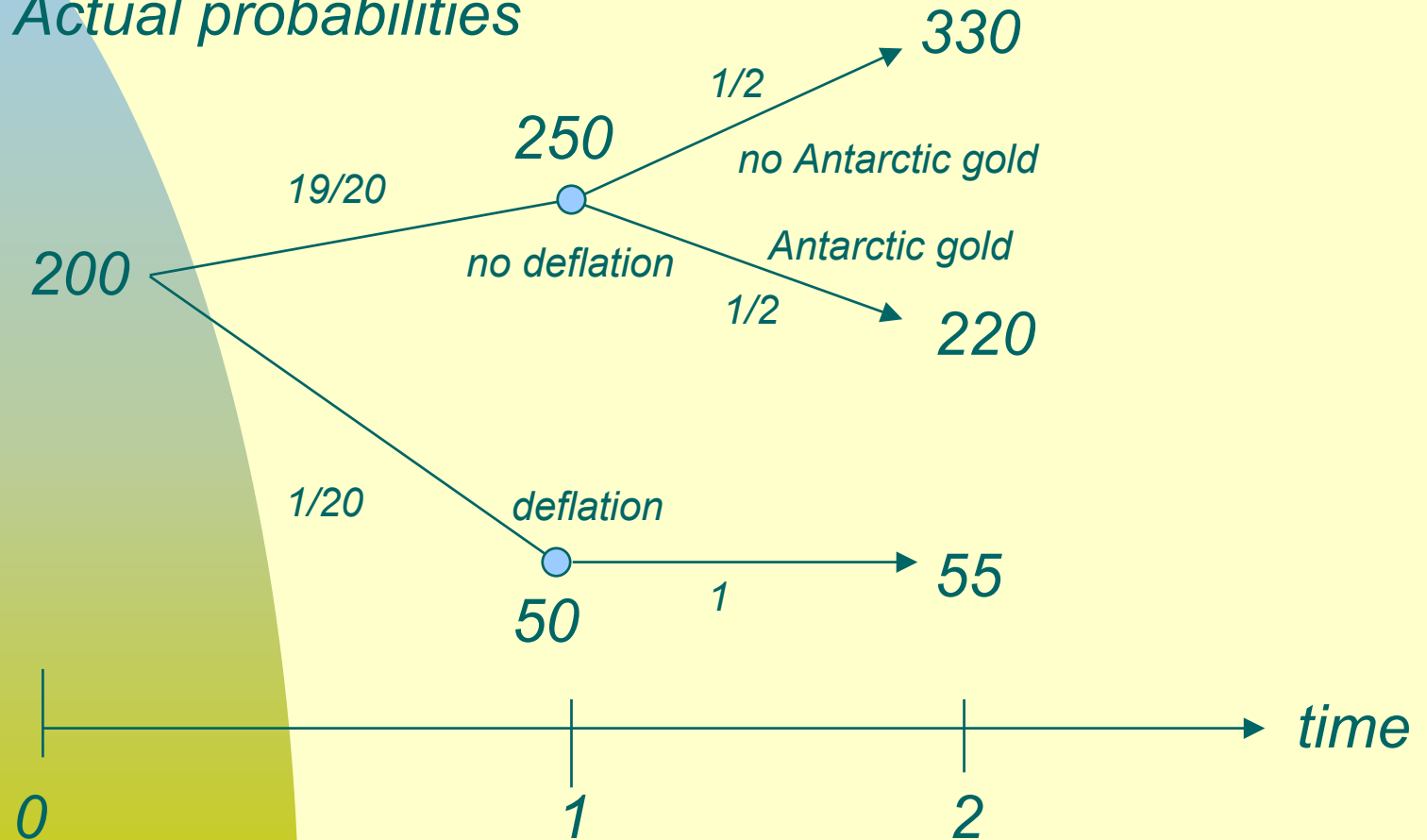
- *Three standard approaches*
 - ◆ *Tree Approach:* With explicit arbitrage arguments using traded assets which are used to construct decision trees
 - ◆ *Continuous Time Approach:* Use Black Scholes theory of rate of return adjustment
 - ◆ *Shoehorn Approach---*Try to find a financial options formula that fits (sort of) the cash flow pattern for your project, use its RA probabilities.

Tree Approach Example

- Facts: Golden rod is considering building a mine:
 - ◆ All the mine's output will be extracted in year 2.
 - ◆ GR can either start building the mine now or build at a leisurely pace or
 - ◆ Start building in year 1.

Tree Approach: Market prices for gold

*Gold prices with
Actual probabilities*



Tree Approach: Market prices

- ◆ The riskfree interest rate on government bonds is 10%
- ◆ gold trades on an organized, efficient market; gold prices will depend on two factors: (ignore storage costs)
 - ☞ whether deflation occurs in the economy
 - ☞ whether a new vein of gold is found in Antarctica

Tree Approach: The investment decision

- ◆ Goldenrod's mine, if constructed will produce 1M oz. of gold. Unit production costs will equal \$100.00 oz.
- ◆ The required investment for the mine equals
 - \$100M paid at date 0---if the mine is constructed at date 0. (NOW)
 - \$120M paid at date 1---if the mine is constructed at date 1. (DELAY)

Tree Approach: the choices

- Goldenrod can follow one of three policies
 - ◆ Start construction of the mine NOW.
 - ◆ DELAY until date 1---construct only if deflation does not occur
 - ◆ ABANDON the mine project

Tree Approach: Valuing the option

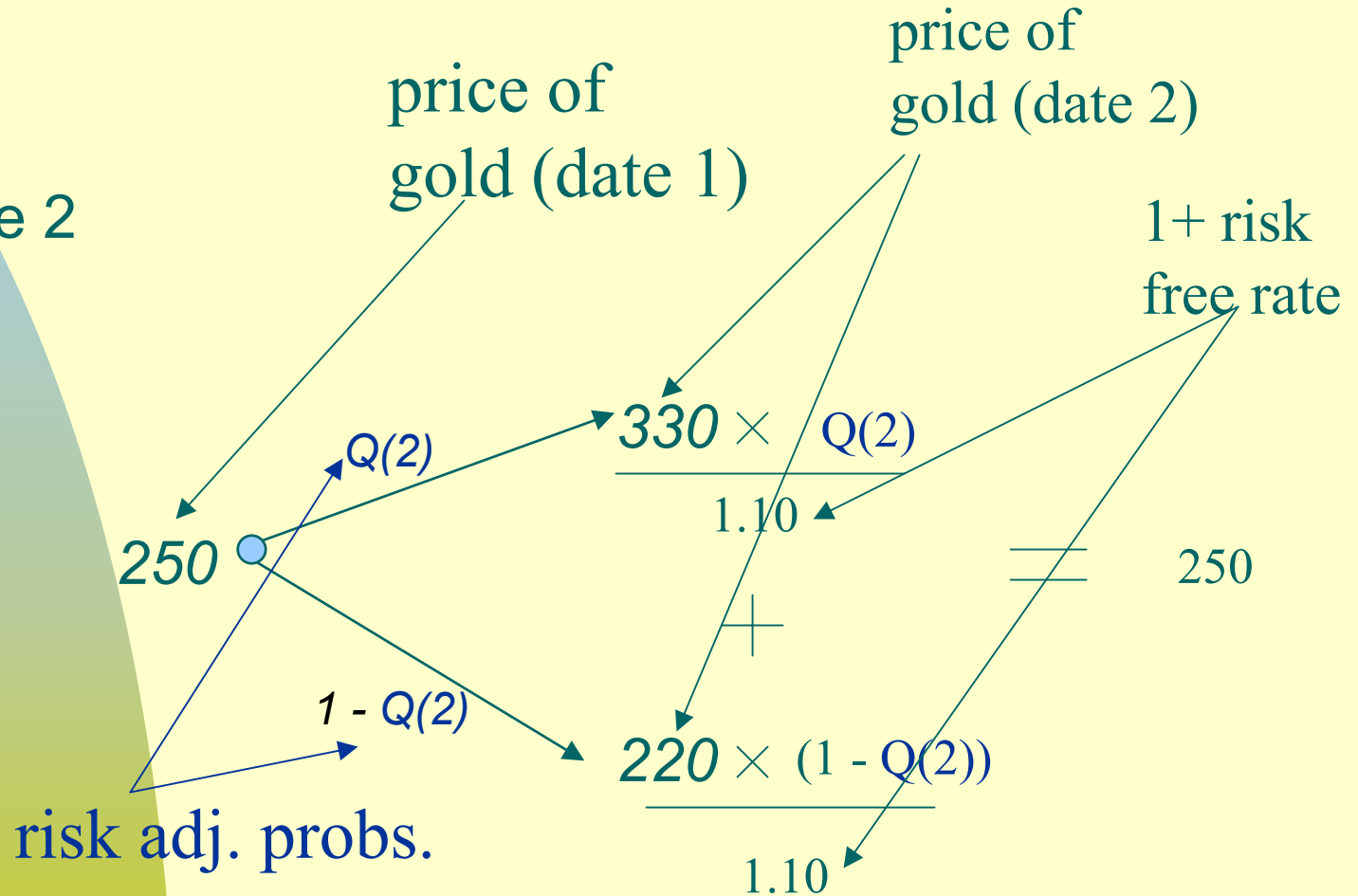
- (1) Construct decision tree
- (2) Use r.n. valuation equation to compute r.n. probabilities.
 - ☞ start at tips of the tree and work backward.
- (3) Use these probabilities to evaluate investment options
 - ☞ “peel the tree back”--solving for the optimal policy starting at the tips of the tree.

Tree Approach: Grinding out the probabilities

- Computing risk-adjusted probabilities
 - ◆ Let $Q(d)$ represent the risk neutral probability of an uptick at date d .
 - ◆ Work backwards through the tree computing $Q(d)$

Tree Approach: the numbers

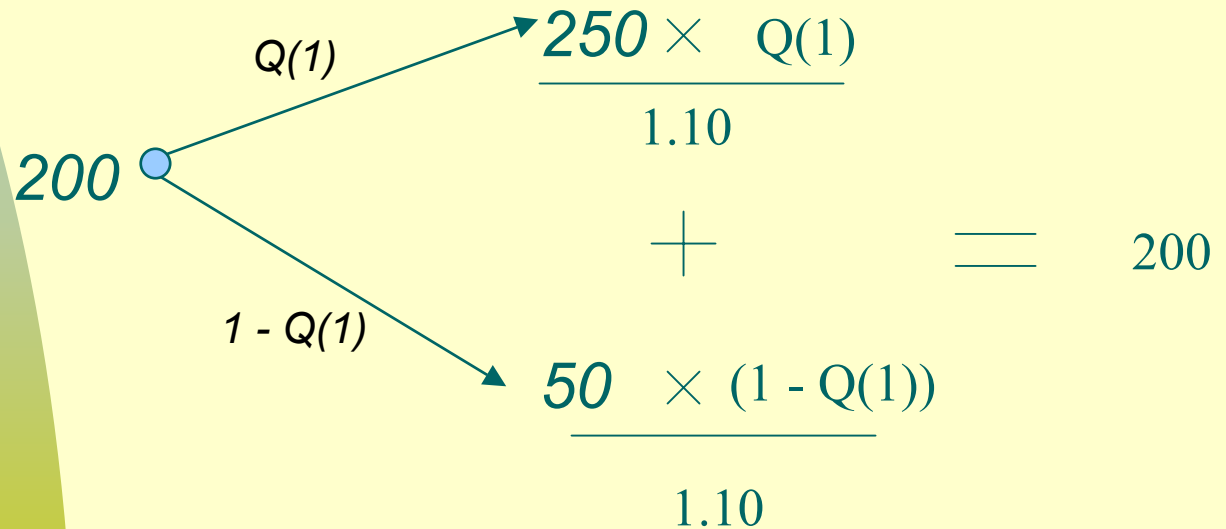
- Date 2



Therefore: $Q(2) = 1/2$

Tree Approach: the numbers

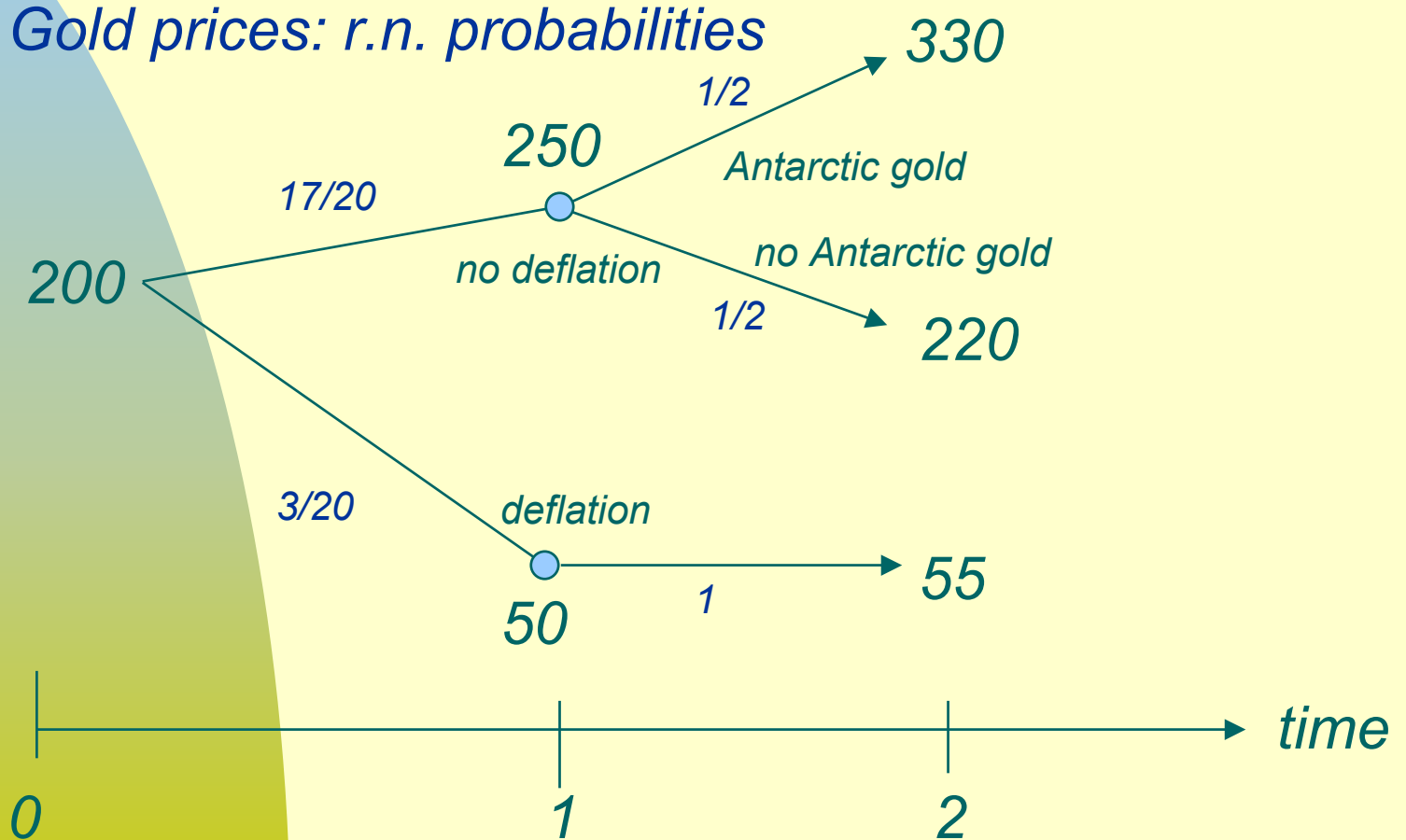
- Date 1:



Therefore: $Q(1) = 17/20$

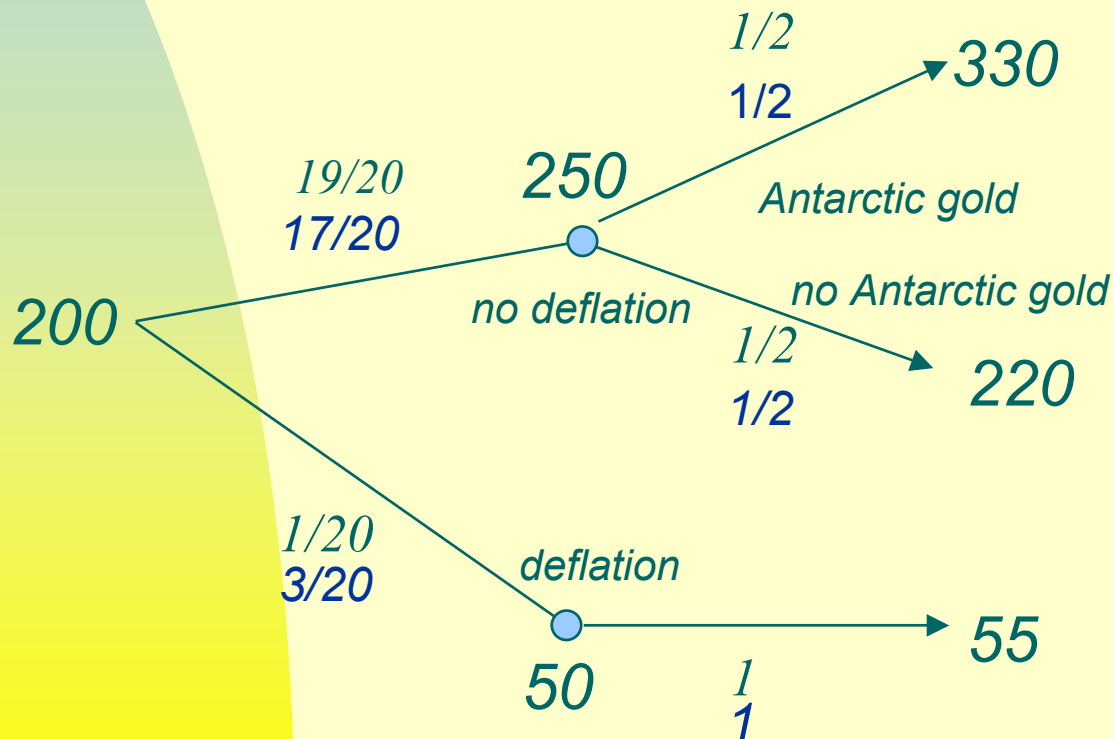
Tree Approach: Risk-neutral probabilities

Gold prices: *r.n.* probabilities



Tree Approach: RA versus actual

- *r.n. probabilities & actual probabilities*



Tree Approach: Value of inflows (r.n. probs)

(1)	(2)	(3)	(4)	(2)X(3)X(4)
<i>goldprice</i>	<i>r.a.prob</i>	<i>ouput</i>	<i>unit margin</i>	
330	17/40	1M	230	97.75
220	17/40	1M	120	51.00
55	6/40	0M	-45	<u>0.00</u>
<i>Exp. CF</i>				148.75
<i>Disc. Rate.</i>				x $1/(1+.10)^2=$
Value of Inflows				122.934

Tree Approach: Value of outlays (r.n. probs)

(1)	(2)	(3)	(4)	(5)	(2)x(4)x(5)
Decision	Outlay	Date	Dis. factor	r.n. prob	Value of Outlay
Now	<i>100</i>	<i>0</i>	<i>1.00</i>	<i>1.00</i>	100.000
Delay	<i>120</i>	<i>1</i>	<i>1/(1.10)</i>	<i>(17/20)</i>	92.7273

Tree Approach: NPV of alternatives

- NPV without delay

$$\begin{aligned} \text{Value of Inflows} - \text{Value of outflows} &= \\ &= 122.934 - 100.00 = 22.934 \end{aligned}$$

- NPV with delay

$$\begin{aligned} \text{Value of Inflows} - \text{Value of outflows} &= \\ &= 122.934 - 92.7273 = 30.2066 \end{aligned}$$

So the option to delay is valuable to Goldenrod

Continuous Time Approach

- Black and Scholes (amongst others) showed that under certain conditions

The expected cash flow from the project
under the risk-adjusted probability distribution

=

the expected cash flow obtained by adjusting the rate of return
on the underlying asset to the risk free rate.

Continuous Time Approach: The Formula

- Continuous time BS real option formula

$$\text{Price}(f(A)) = e^{-rT} E_P[f(A^{\text{rf}})]$$

f is the relationship between asset value and project value

A^{rf} is the value of the underlying asset under the risk adjusted value process

r is the risk free rate of interest

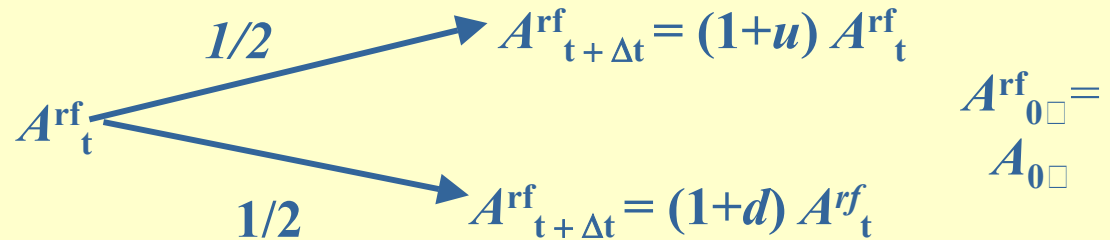
Continuous Time Approach: The Formula

- Next compute expectations with Monte Carlo Simulation of

$$e^{-rT} f(A^{\text{rf}})$$

- Average the values from the simulations

Risk Adjusted Asset Value Process: A^{rf}



$$u = r \Delta t + \sigma \sqrt{\Delta t}, \quad d = r \Delta t - \sigma \sqrt{\Delta t}, \quad \Delta t \approx 0$$

$$dA_{rf} = (r dt + \sigma dB) A_{rf}$$

B--standard (0,1) Brownian motion
(Wiener process)

σ -- is the instantaneous volatility
of the assets value

Shoehorn Approach: Example

- Try to fit you real options problem into the formula for a textbook financial options problem.

- ◆ E. g., Consider the peaker plant problem: Value

$$\text{Max}[E_T - h \times F_T - c, 0]$$

Shoehorn Approach: Example

- We can map this problem into a text book problem

Peaker plant problem: Value

$$\text{Max}[E_T - h \times F_T - c, 0]$$

Financial Option Exchange Problem:
Value:

$$\text{Max}[S1 - S2, 0]$$

- Use the textbook solution: Margrabe exchange formula (1978, Journal of Finance)

Financial vs. Real Assets



Financial assets can be costlessly stored and traded (almost!)

Real assets are sometimes hard to store and costly to trade.

Prices of real assets behave differently than financial assets



Financial vs. Real Assets

- Problem: Valuation of real investment opportunities for energy (commodity) firms
- In principle: Nothing Changes!!
 - ◆ Basic principles are unchanged:
 - ☞ That is, if the cash flows from the market can be replicated by a traded financial asset, the same real-option pricing develop in the last lecture works.

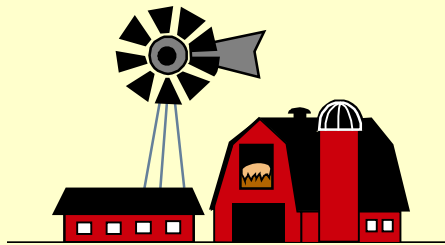
Financial vs. Real Assets

- In practice: Valuation problem is tricky
 - ☞ Financial markets required Don't exist
 - ☞ On the other hand,
 - However, significant
 - Related markets and
 - Market-based information

Do Exist!

These markets can be used to help price real options associated with energy investments.

Storable commodity



No assets tracking the financial asset.
However asset can be stored.

Storable commodities

- Suppose:
- financial forwards and/or spot price securities do not trade.
- However, commodities are storable,
- Can we apply no arbitrage arguments to the spot-price series, $p(\cdot)$, to value real options.

Storable commodities

- We can apply arbitrage arguments to the commodity price series under the following set of assumptions
 - ◆ The commodity can be bought and sold in a frictionless market.
 - ◆ Short positions (i.e. borrowing the commodity) are feasible.
 - ◆ Holding the commodity produces a constant instantaneous convenience yield

Storable commodities

- ◆ The “convenience yield” can be either positive or negative
 - ☞ Positive convenience yield, e.g., commodity throws off cash flows (farmland, a developed oil field)
 - ☞ Negative convenience yield---e.g., commodity is costly to store and throws of no positive cash flows (e.g. an antique).

Storable commodities

- Spot price process used for pricing storable commodity

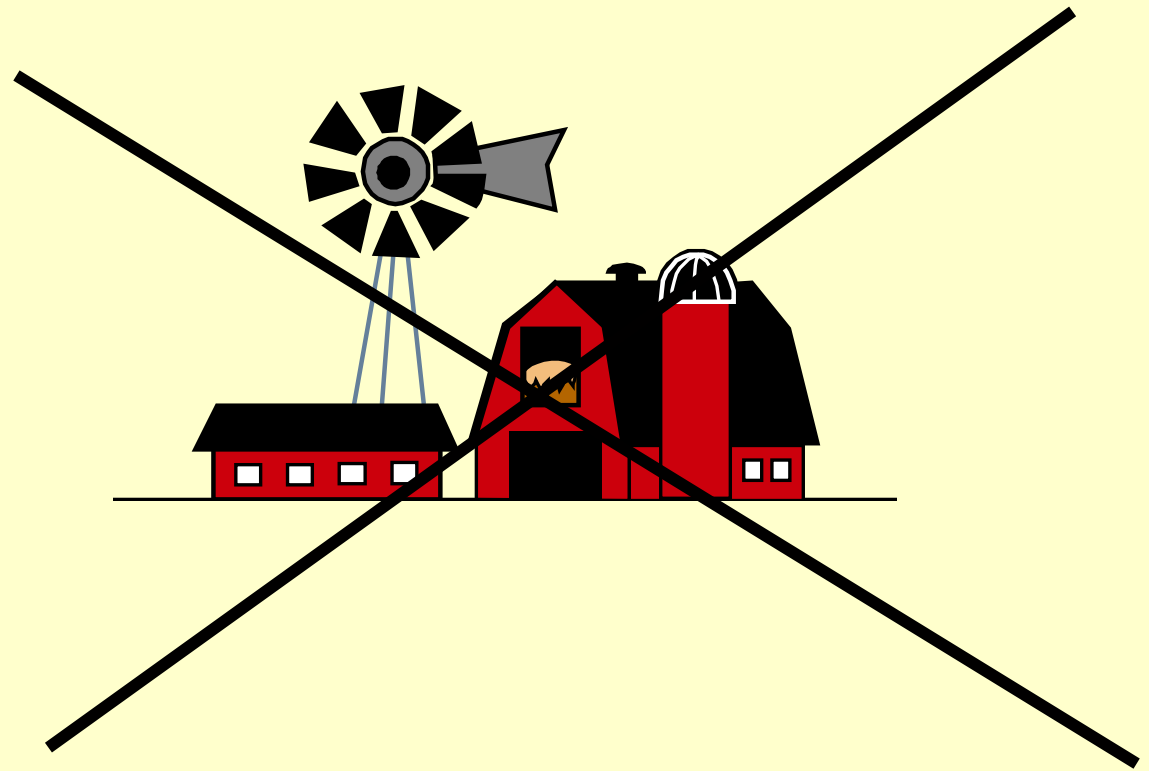
$$C_{t+\Delta t} = \begin{cases} (1 + (r - \delta)\Delta t + \sigma\sqrt{\Delta t})C_t & \text{with prob. } 1/2 \\ (1 + (r - \delta)\Delta t - \sigma\sqrt{\Delta t})C_t & \text{with prob. } 1/2 \end{cases}$$

Δt infinitesimal, or in another representation of the same idea:

$$dC = ((r - \delta)dt + \sigma dB)C$$

same old model as before but with a convenience yield added

Non-storable commodity



Modeling non-storable commodities

- Example: Electricity
- Electricity Cannot be stored and
 - ◆ Spot prices (in short-run) depend on air temperature
 - ◆ Temperature is
 - ☞ (a) seasonal (what a revelation!)
 - ☞ (b) highly mean-reverting
(If its 110 F, in March in NO, tomorrow will be cooler, if its 40 F, tomorrow will be warmer)

Modeling non-storable commodities

- Therefore spot price
 - ◆ Will never fit risk-adjusted stock price model
 - ◆ Therefore need to use
 - ☞ Financial forwards and futures contracts on power to price assets OR
 - ☞ Adjust spot prices for mean reversion before using volatility estimates from spot prices to price assets

Summary

- .Real Option Valuation
 - ◆ What: Takes into account
 - ☞ Project optionality
 - ☞ Dependence of cost of capital for a project on the fine structure of the project
 - ◆ How: risk adjusted probability distribution
- Application: requires estimating the value process of an underlying traded financial asset.