Mathematical Models for Post-Disaster Reconstruction Planning in Urban Areas

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Goal

Provide reconstruction framework by redrawing boundaries in order to assign appropriate land use to the neighborhoods.

Hurricane Katrina

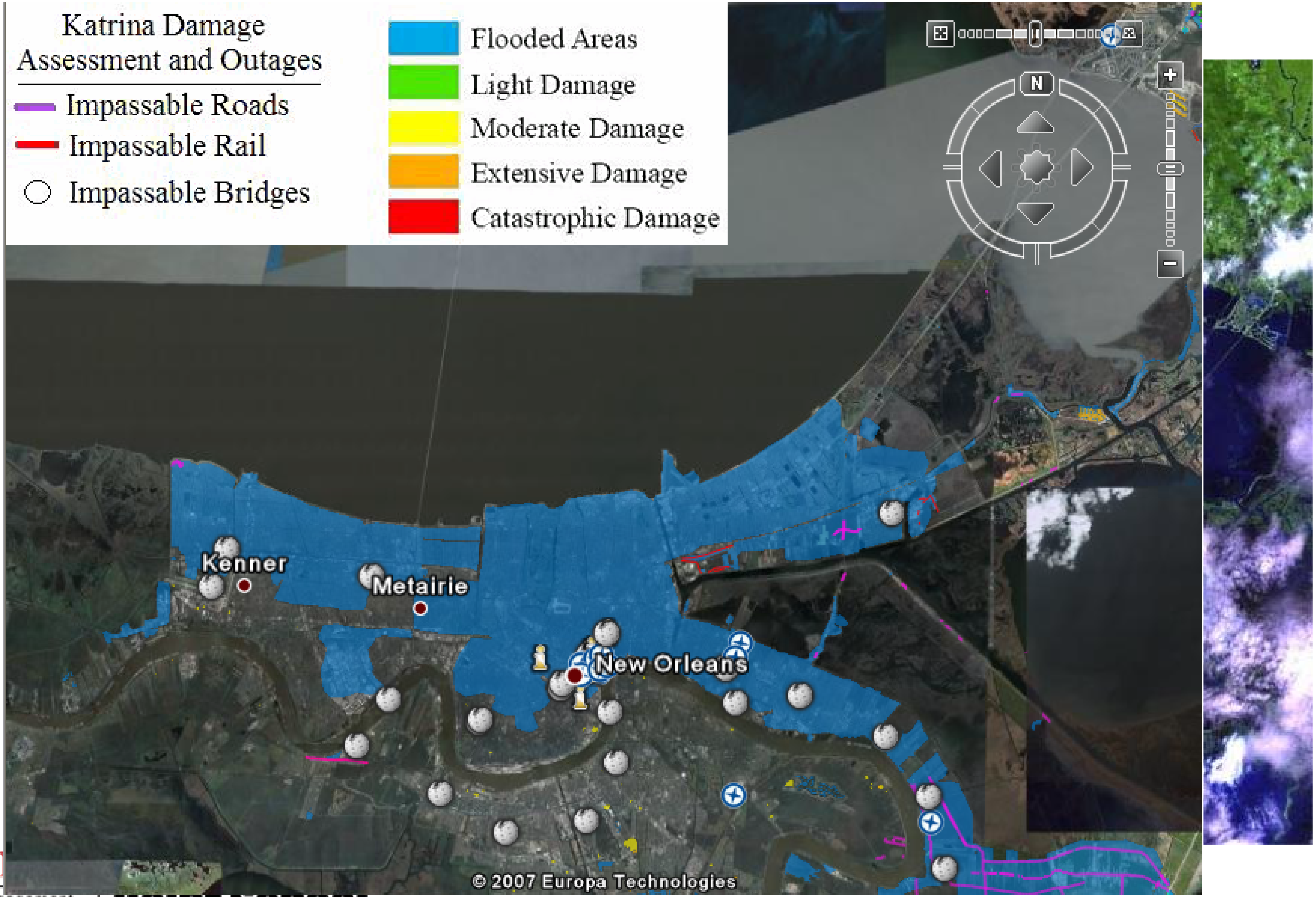
- The third strongest landfalling U.S. hurricane, and the costliest in U.S. history.
- Deadliest since 1928 Okeechobee
- Louisiana is severely damaged

	Louisiana	Total
Damaged Home	515,249	1,197,499
Severe Damage	106,651	125,731

Source: FEMA Gulf coast housing damage estimates

Deaths by state	
Alabama	2
Florida	14
Georgia	2
Kentucky	1
Louisiana	1,577
Mississippi	238
Ohio	2
Total	1,836
Missing	705

What happened in New Orleans



Map source: NGA (National Geospatial Agency)

Why to redevelop

- Historically, not many cases!
- Emotional cost (Smith 2001, Hartman and Squires 2006)
- Economic, historical importance - $500 billion in real estate assets, 19 districts (38,000 properties) in National Register of Historical Places.

Yes, we should redevelop! But the question is not 'whether or not redevelop Simpson's neighborhood.'

It is 'How to redevelop Simpson's city in order to provide him a better place to live!'

Planning is important

- Reconstruction phase takes 10 times longer than emergency period and restoration period (Kates *et al*, 2006).
- The cost of natural disaster reconstruction is especially high in urban areas (Boulle *et al* 1997).

OR/MS can add value to reconstruction planning

- Competing objectives
- Traditional boundaries are insufficient.
- MCDM (Opricovic *et al* 2002) can solve with 2.43647×10^{15} alternatives?
- Optimization method has not been used.

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Research Questions

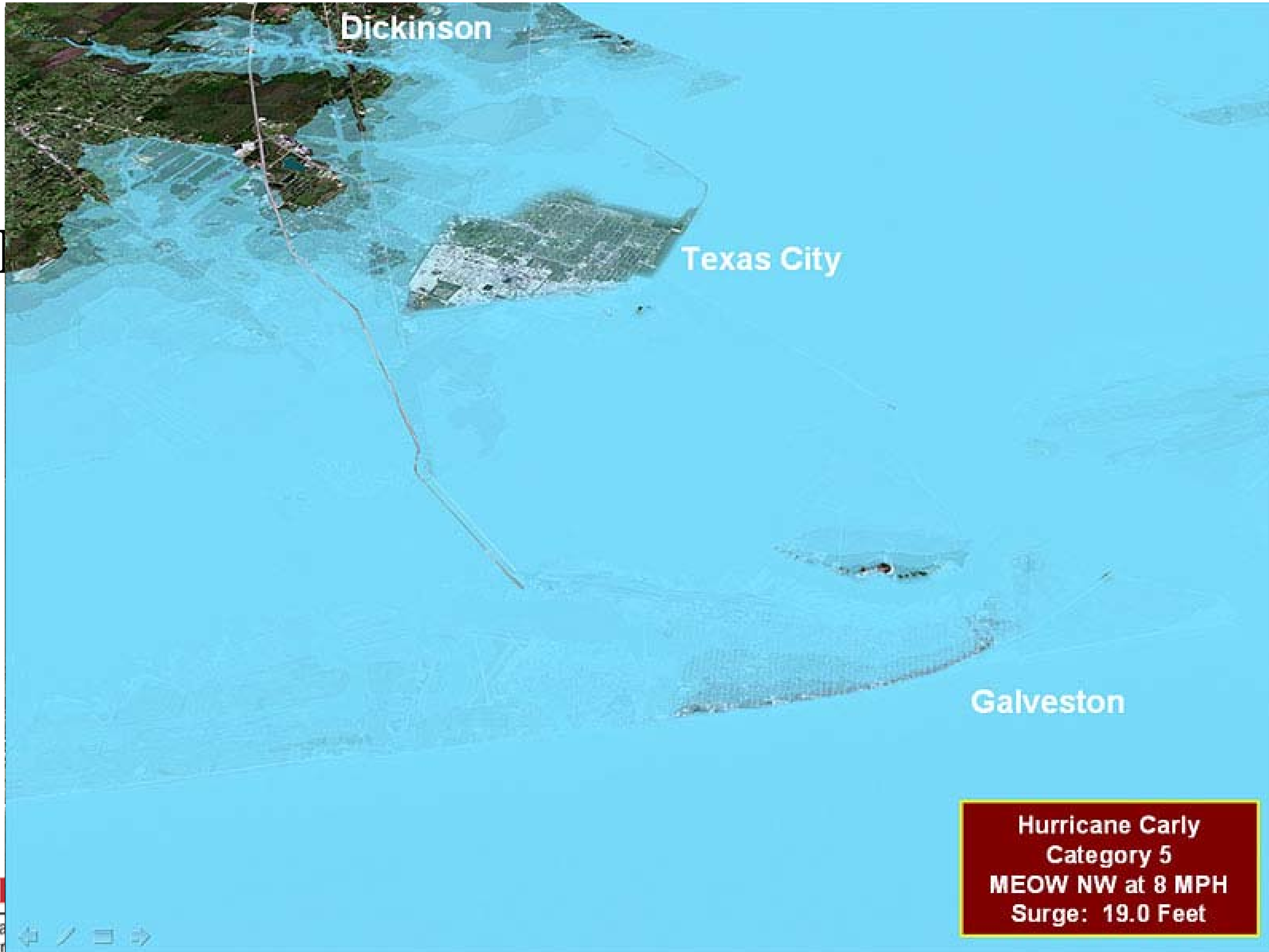
- Can we assist planners to make better decisions regarding difficult problems by providing a planning framework using mathematical programming?
- Is there a viable alternative to the mathematical programming?

Objectives

Key concerns

1. Distance between water source and land parcels
2. Elevation of parcels
3. Contiguity/compactness of parcels
4. Social Impact of parcels

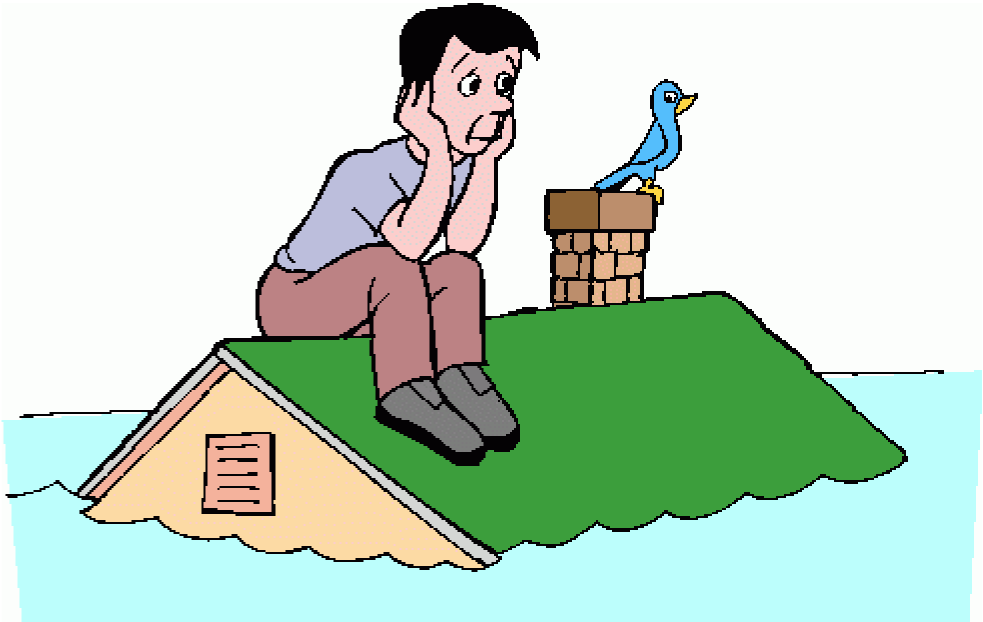
1. Distance to water source



2. Elevation

- 100-year flood plain is based on Elevation
- NFIP uses Elevation for insurance assessment.
- 3-foot rule is insufficient.

NFIP: National Flood Insurance Program by FEMA



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3. Contiguity/Compactness

- Minimize negative impact from outside (Williams et al 2005, Wright et al 1983).
- Agriculture: Isolated farm brings more complaints from neighbors and has negative impact. (Bryant and Johnston 1992)
- Economies of scale exist. (construction, infrastructure, services)

4. Social Impact (SI)

- Measure the socio-economic status.
  - Derived from Population, Income, number of open schools, distance to the operating hospitals.
- Support sustainable development.
  - Do not abandon 70% of the city!

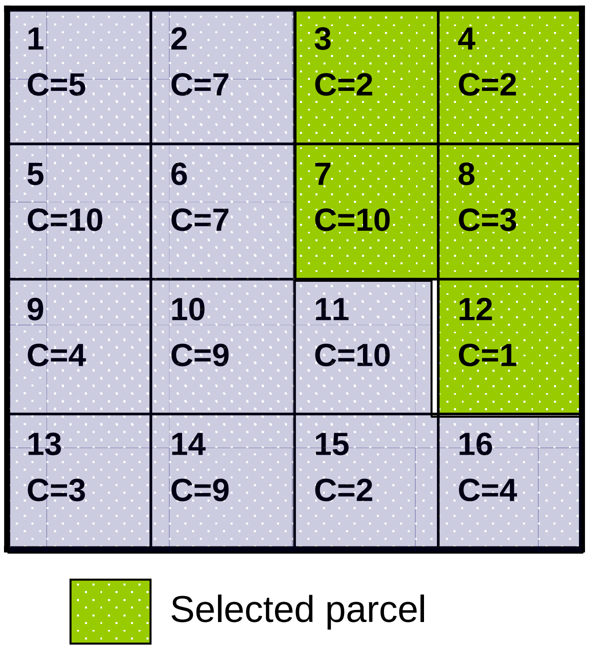
OR Approaches

Alternative Modeling Approaches

- Reserve design model
  - Choose land parcels to reserve for nature/endangered species protection.
- Land allocation/acquisition
  - Select an area of land for development
- Political districting/districting
  - Find an optimal plan to partition an area into its sub-areas such that each sub-area's population is nearly equal.

Reserve Design

- Reserve design model (Williams, ReVelle, and Levin, 2005)



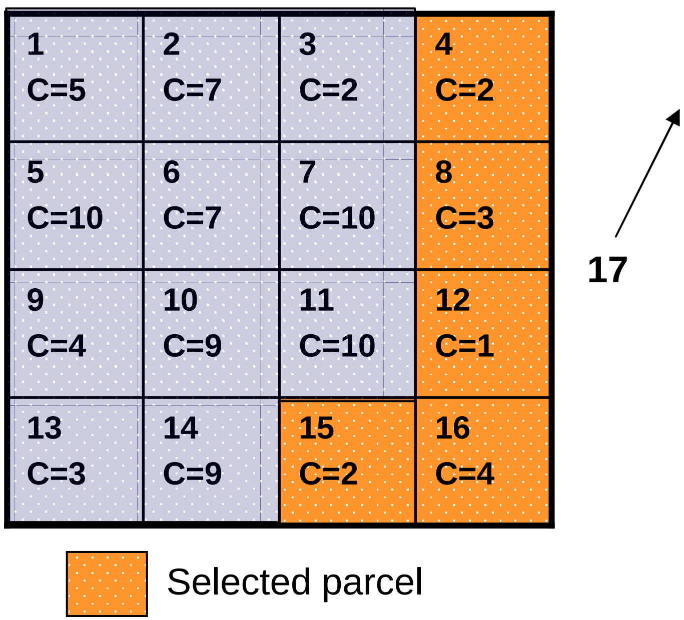
Weight on distance = 0.5
Weight on connectivity = 1
Weight on compactness = 0.5
Weight on cost = 0.1

What about
the rest?

Land Acquisition

- Land acquisition model (Wright, ReVelle, and Cohon, 1983)

Weight on cost = 0.5
Weight on compactness = 0.5

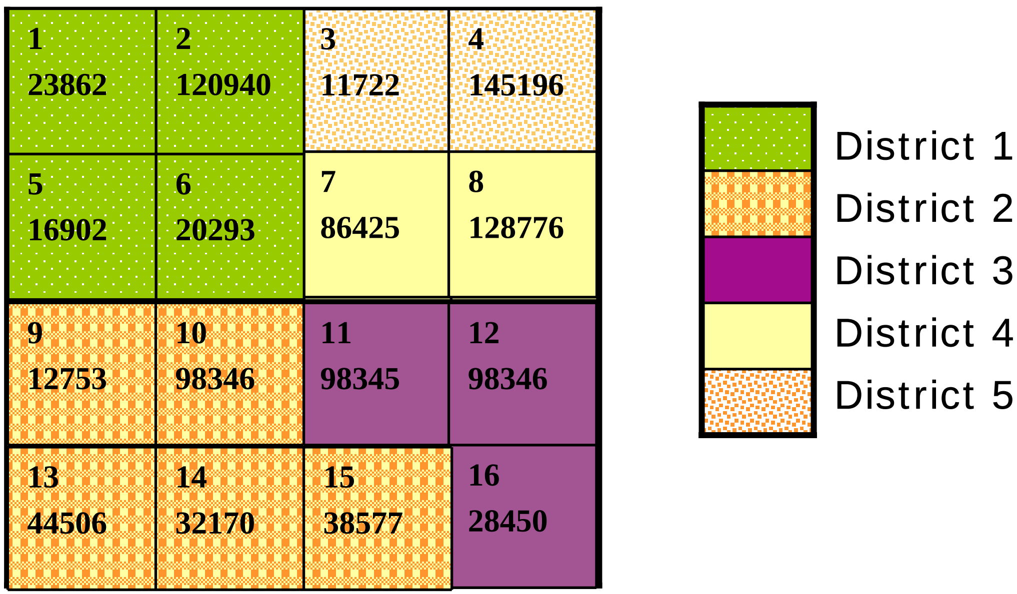


The outer part of entire region is regarded as one big parcel (irregular shape), and the parcel number is 17.

What about the rest?

Political Districting

- Political Districting model (Hess *et al*, 1965)



Numbers in each parcel: population of the parcel
With max allowable population deviation = 25%
M = 5 districts
Data: Mehrotra et al (1998) –counties in South Carolina

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Comparison

- Reserve design and Land allocation do not account for the unselected parcels.
- Political districting problem does not capture the characteristics of parcels, but its partitioning is suited for our purpose.

How can we do better

- Multiobjective Integer Programming
  - Multiple goals – distance, elevation, connectivity, and SI
- Modified Political Districting problem
- Data from New Orleans
- Test using AMPL/CPLEX (10.0.0)

AMPL is a programming language widely used in OR/MS.
CPLEX is software that solves the optimization problem.

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Planning Model

Modeling Frame

- Identically-sized parcels
- Land use types: Human habitat (residential, commercial area) and Passive use (wetland, park, etc.)
- Economies of scale exist in reconstruction project.
- The higher the elevation, the better for human habitation.
- Parcels in the same districts are assigned to the same land use.
- Fair Social Impact (Broadmoor's protest)

Model Description

- Optimize
  - Minimize the distance to water from Passive use parcels.
  - Maximize the elevation of human use parcels.
  - Minimize the external borders.
  - Maximize Social Impact of human use parcels.
- Subject to
  - Each parcel is assigned to one and only one district and land use type.
  - Parcels in the same district are assigned to the same land use.
  - Social Impact of a district falls in allowable range.

Notation

Set

$i =$ Index of parcels ($i = 1, 2, ..., n$)

$j =$ Index of districts ($j = 1, 2, ..., D$)

$k =$ Index of the land use type ($k =$ "H" human habitat, $k =$ "P" passive use)

Parameters

$D =$ The number of districts required

$D_p =$ The number of districts of passive land use

$C =$ Amount of Budget

$\alpha =$ percentage of allowable deviation of social benefit from average social benefit

$d_i =$ Euclidean distance from the water source to a neighborhood i

$e_i =$ Elevation level of a neighborhood i

$c_{ik} =$ Cost of reconstruction in parcel i for landuse k

$s_i =$ Net social benefit of neighborhood i

$M =$ Big M (= total number of parcels)

$A_i =$ Set of parcels which are adjacent to parcel i

$w_1 =$ Weight on the distance objective

$w_2 =$ Weight on the elevation objective

$w_3 =$ Weight on the connectivity objective

$w_4 =$ Weight on the net social benefit objective

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Decision Variables

$x_{ijk} = 1$ if the i^{th} parcel is assigned to the j^{th} district and land use k, 0 otherwise.

$B_j = 1$ if the j^{th} district is to be developed for Passive use, 0 otherwise.

$P_{ilj} = 1$ if $\sum_k x_{ijk} = 1$ and $\sum_k x_{ljk} = 0$, 0 otherwise.

(1 if parcel i is included in district j, and l is not, 0 otherwise)

$N_{ilj} = 1$ if $\sum_k x_{ijk} = 0$ and $\sum_k x_{ljk} = 1$, 0 otherwise.

(1 if parcel l is included in district j, and i is not, 0 otherwise)

Minimize $w_1 \sum_{i=1}^{n} \sum_{j=1}^{D} d_i x_{ijP} - w_2 \sum_{i=1}^{n} \sum_{j=1}^{D} e_i x_{ijH} + w_3 \sum_{i=1}^{n} \sum_{l \in A_i} \sum_{j=1}^{D} (P_{ilj} + N_{ilj}) - w_4 \sum_{i=1}^{n} \sum_{j=1}^{D} s_i x_{ijH}$

Distance between water source and passive use parcels

Elevation of human use areas

Total length of External borders

Total SI of human use

Subject to

$$\sum_{j=1}^{D} \sum_{k} x_{ijk} = 1 \qquad \forall i$$

$$\sum_{i} x_{ijP} \geq \sum_{i} \sum_{k} x_{ijh} - (1 - B_j) \times M \qquad \forall j$$

$$\sum_{i} x_{ijH} \geq \sum_{i} \sum_{k} x_{ijk} - B_j \times M \qquad \forall j$$

$$\sum_{i=1}^{n} s_i x_{ijk} \geq (1 - \alpha) \sum_{i=1}^{n} \sum_{d} \frac{s_i}{D} \qquad \forall j$$

$$\sum_{i=1}^{n} \sum_{k} s_i x_{ijk} \leq (1 + \alpha) \sum_{i=1}^{n} \sum_{d} \frac{s_i}{D} \qquad \forall j$$

$$\sum_{k} x_{ijk} - \sum_{k} x_{ljk} - P_{ilj} - N_{ilj} = 0 \qquad \forall i, \forall l \in A_i, \forall j$$

$$\sum_{i} \sum_{j} \sum_{k} c_{ik} x_{ijk} \leq C$$

$$\sum_{j=1}^{D} B_j = D_p$$

$$\sum_{i} \sum_{k} x_{ijk} \geq 1 \qquad \forall j$$

$$x_{ijk}, P_{ilj}, N_{ilj}, B_j : binary$$

Problem Analysis

- The problem size (D.V. and Const.) grows in $O(n^2)$ where n is the number of parcels.
- NP-hard – hard to solve.
- In reality, we may need 10^2 parcels
- Alternative – Local search heuristic method
- IP vs. heuristic?

IP: Integer Programming

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Heuristic Algorithm

- Normalized data: distance, elevation, SI

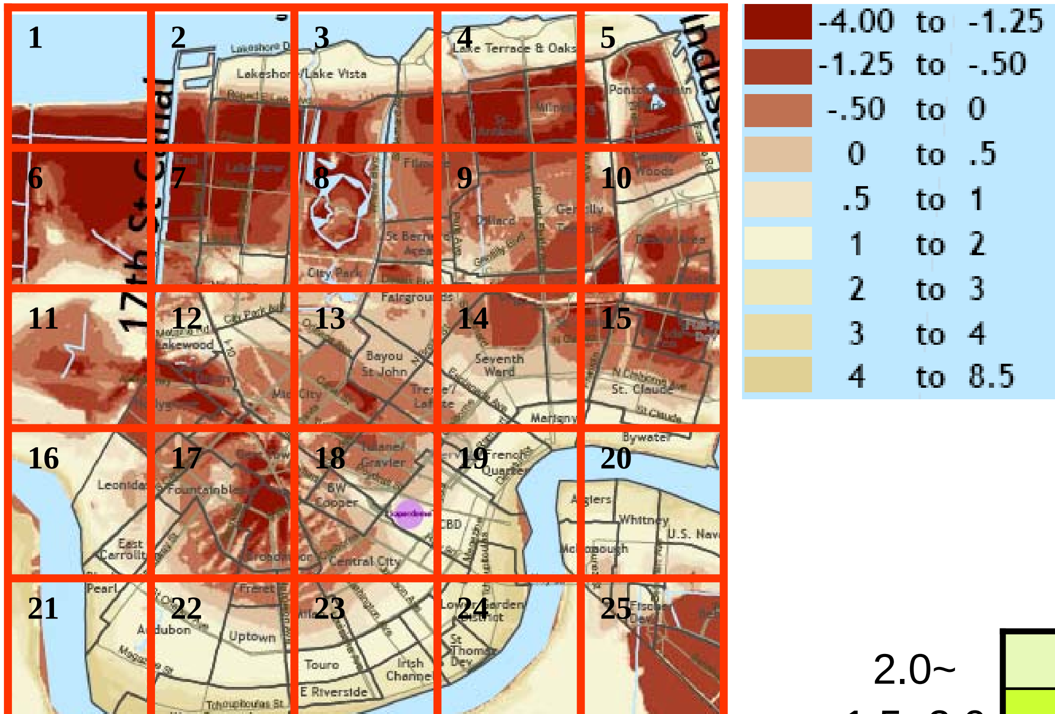
$$\bar{e}_i = \frac{e_i - e_{\min}}{e_{\max} - e_{\min}} \quad \text{for "H"} \qquad \underline{e}_i = \frac{e_{\max} - e_i}{e_{\max} - e_{\min}} \quad \text{for "P"}$$

- Normalized H/P-achievement = Normalize sum(normalized data)
  - Start with the best H-achievement and add adjacent parcel to compose a district.
  - Each time perform enclave check (Nemhauser and Garfinkel 1970) : if enclaved, backtrack.
  - Stop if the total SI falls in allowable range.
- Running time is $O(n^3)$

DATA

New Orleans Data

New Orleans elevation



Source: USGS (US Geological Survey)

SI estimation

$$SI_j = Population_j + Income_j + School_j + Hospital_j$$

Population, Income: pre-Katrina
Open School, Distance to operating Hospital: post-Katrina

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Data Summary

Distance (mile)		Elevation (m)		SI		Cost
measured	value used	measured	value used	measured	value used	Random
0.7 ~ 1.6	1	-4 ~ -1.25	-4	0 ~ 0.5	1	Discrete value ranging between 1 and 10
1.7 ~ 3	2	-1.25 ~ -0.5	-3	0.5 ~ 1.0	2	
3.0 ~ 4.5	3	-0.5 ~ 0	-2	1.0 ~ 1.5	3	
		0 ~ 0.5	-1	1.5 ~ 2.0	4	
		0.5 ~ 1	0	2.0 ~	5	
		1 ~ 2	1			
		2 ~ 3	2			
		3 ~ 4	3			
		4 ~ 8.5	4			

- Reduce computation cost!

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Test Results

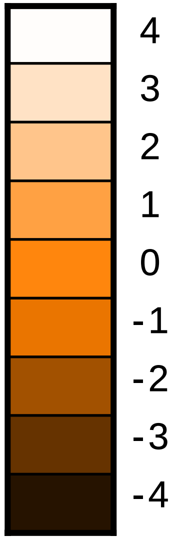
Test Result from Equal Weights

Water Resource - Lake

1 -4	2 -4	3 -3	4 -4	5 -3
6 -2	7 -1	8 -1	9 -2	10 -3
11 0	12 -3	13 -2	14 1	15 1
16 2	17 -1	18 -1	19 2	20 3
21 3	22 2	23 2	24 2	25 -1

Water Resource - River

Elevation level



w1 = w2 = w3 = w4 = 1

Water Resource - Lake

1 P	2 P	3 P	4 P	5 P
6 P	7 P	8 P	9 P	10 P
11 H	12 H	13 H	14 H	15 H
16 H	17 H	18 H	19 H	20 H
21 H	22 H	23 H	24 H	25 H

Water Resource - River

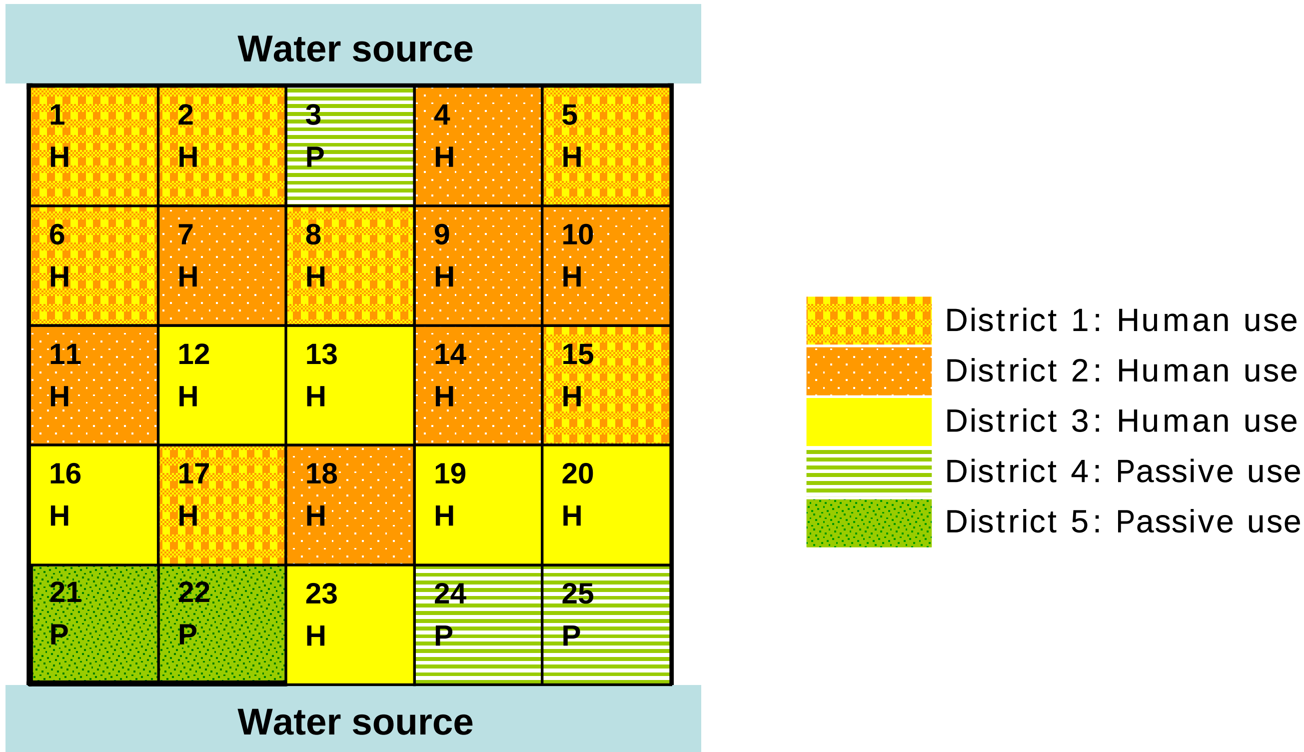
Shaded area is assigned for Passive land use

Total	Dist.	Elev.	Com.	NSB
-13	15	10	26	44

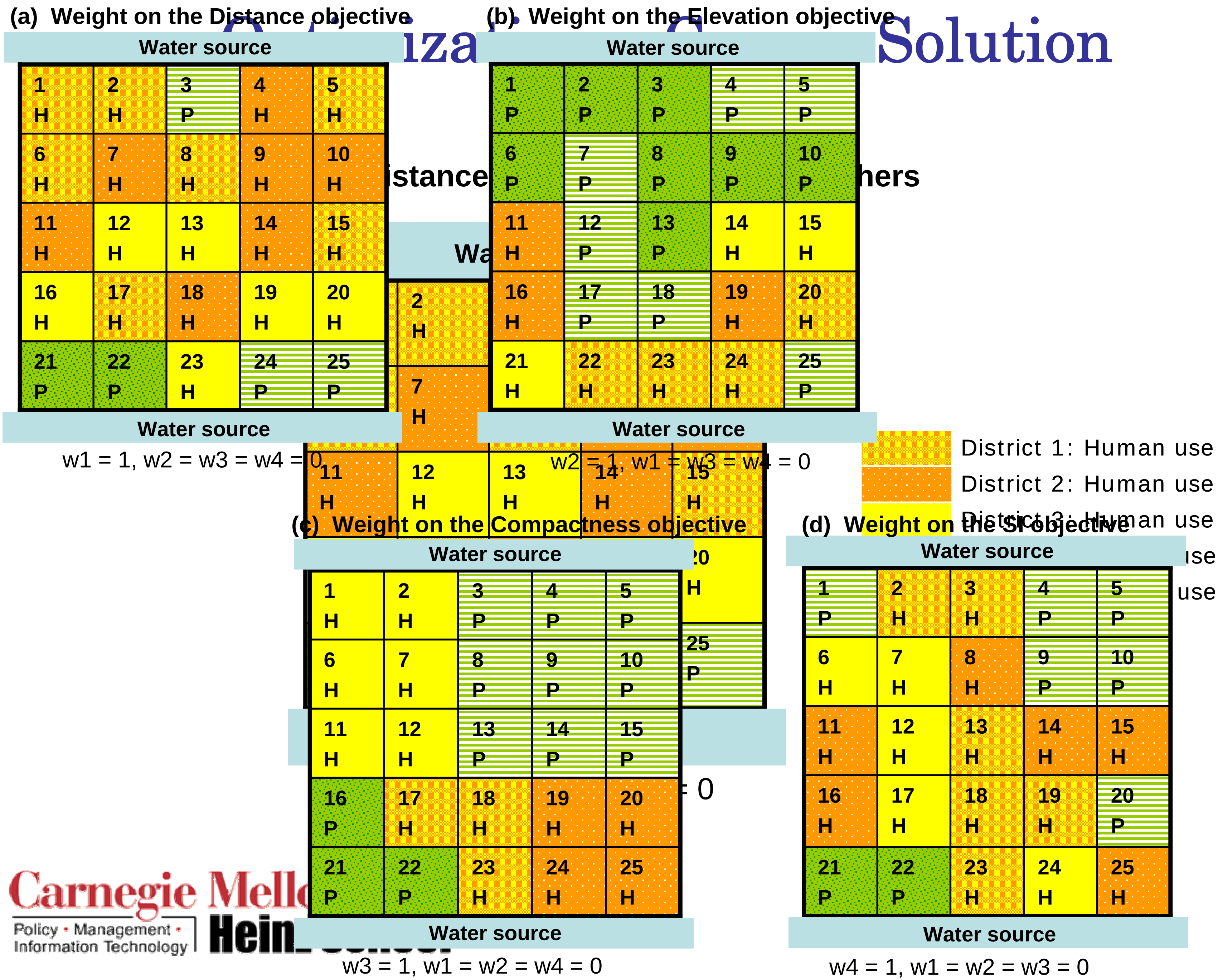
District 1: Human use
District 2: Human use
District 3: Human use
District 4: Passive use
District 5: Passive use

Optimization – Corner Solution

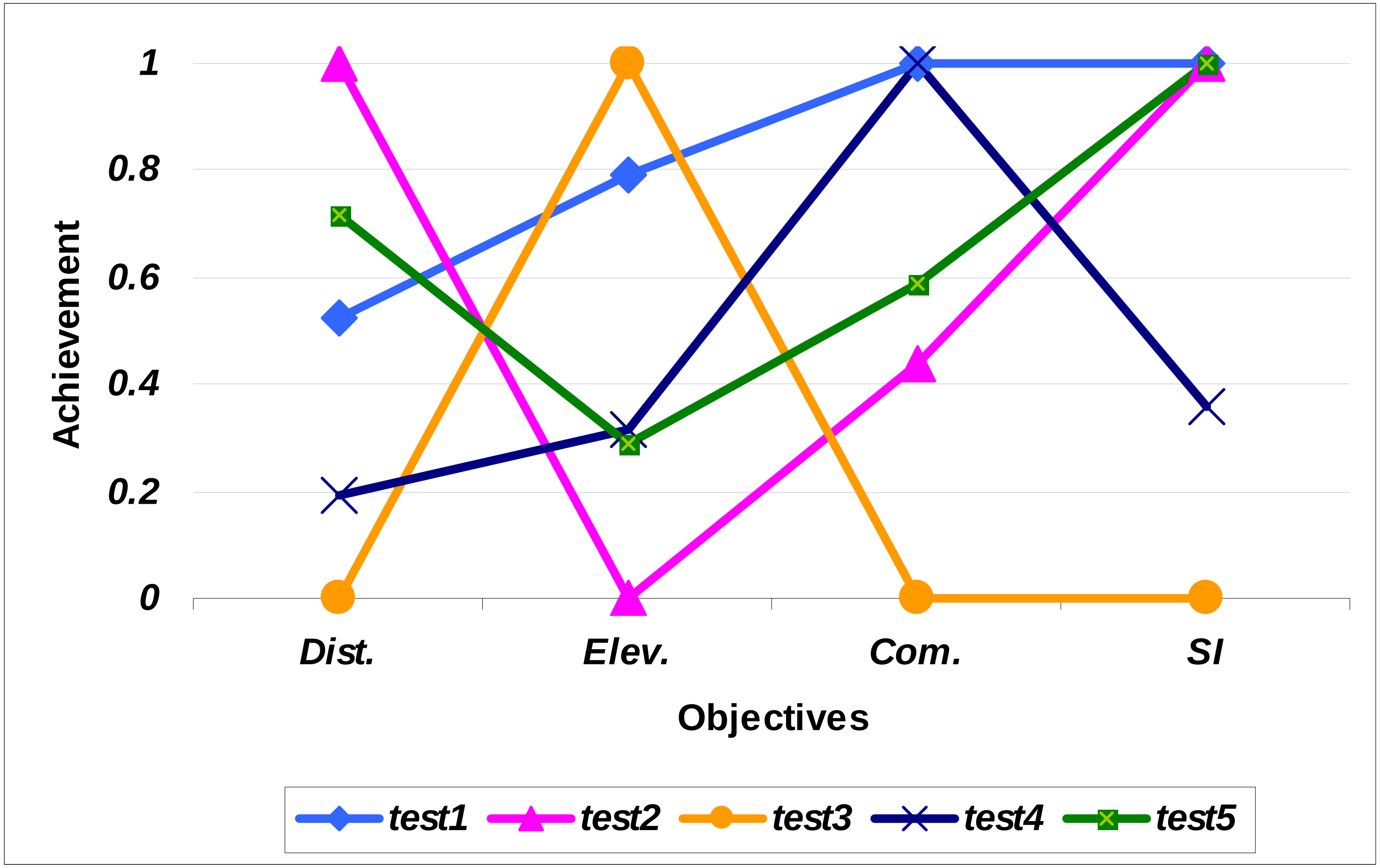
Weight 1 on distance objective, and zero on others



$w1 = 1, w2 = w3 = w4 = 0$


Solution
(a) Weight on the Distance objective
Water source
1 H, 2 H, 3 P, 4 H, 5 H
6 H, 7 H, 8 H, 9 H, 10 H
11 H, 12 H, 13 H, 14 H, 15 H
16 H, 17 H, 18 H, 19 H, 20 H
21 P, 22 P, 23 H, 24 P, 25 P
Water source
w1 = 1, w2 = w3 = w4 = 0
(b) Weight on the Elevation objective
Water source
1 P, 2 P, 3 P, 4 P, 5 P
6 P, 7 P, 8 P, 9 P, 10 P
11 H, 12 P, 13 P, 14 H, 15 H
16 H, 17 P, 18 P, 19 H, 20 H
21 H, 22 H, 23 H, 24 H, 25 P
Water source
w2 = 1, w1 = w3 = w4 = 0
District 1: Human use
District 2: Human use
District 3: Human use
(c) Weight on the Compactness objective
Water source
1 H, 2 H, 3 P, 4 P, 5 P
6 H, 7 H, 8 P, 9 P, 10 P
11 H, 12 H, 13 P, 14 P, 15 P
16 P, 17 H, 18 H, 19 H, 20 H
21 P, 22 P, 23 H, 24 H, 25 H
Water source
w3 = 1, w1 = w2 = w4 = 0
(d) Weight on the SI objective
Water source
1 P, 2 H, 3 H, 4 P, 5 P
6 H, 7 H, 8 H, 9 P, 10 P
11 H, 12 H, 13 H, 14 H, 15 H
16 H, 17 H, 18 H, 19 H, 20 P
21 P, 22 P, 23 H, 24 H, 25 H
Water source
w4 = 1, w1 = w2 = w3 = 0

All solutions are Non-Dominated



test1: unity weight (1, 1, 1, 1) on each obj.
test2: weight on distance obj.
test3: weight on elevation obj.
test4: weight on compactness
test5: weight on NSB of parcels for human land use

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IP Computational Result

Test	No.	w_1	w_2	w_3	w_4	Run time	Obj.	Dist.	Elev.	Comp	NSB	Iteration	B-B
RHS fractional	1	1	1	1	1	24.52	-3	21	13	28	39	169772	5207
	2	0	0	1	0	5252.39	28	9	-26	28	37	6917	122
RHS rounded & Constraint coefficient scaled down to 1~5	3	1	1	1	1	9.25	-13	15	10	26	44	47214	631
	4	1	0	0	0	0.27	5	5	-20	64	44	344	4
	5	0	1	0	0	0.09	-18	26	18	94	30	28	0
	6	0	0	1	0	1018.59	26	22	-8	26	35	6472501	146690
	7	0	0	0	1	0.30	-44	11	-9	54	44	333	7
	8	3	1	1	1	15.14	15	7	-6	32	44	123038	4144

1: Give more weight on feasibility than optimality
2: Cover cut option is likely to be used
3: Gomory fractional cut is likely to be used
4: GUB cut is likely to be used
5: Do not use any cuts

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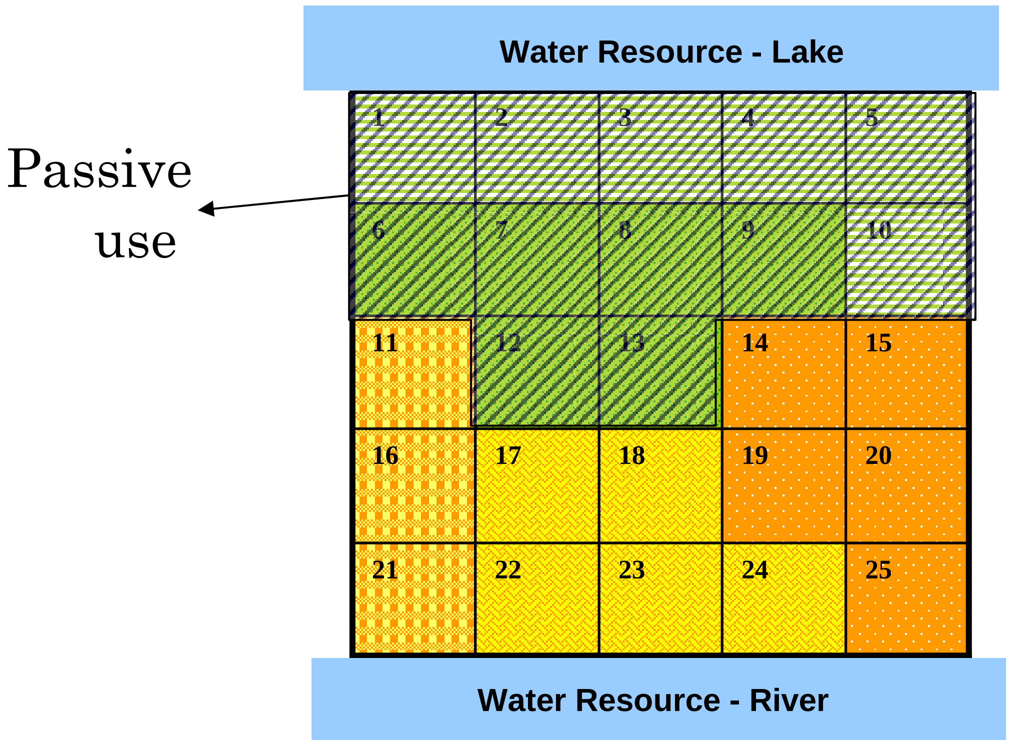
Heuristic Solution is a Promising Alternative



Alpha = **0.25**

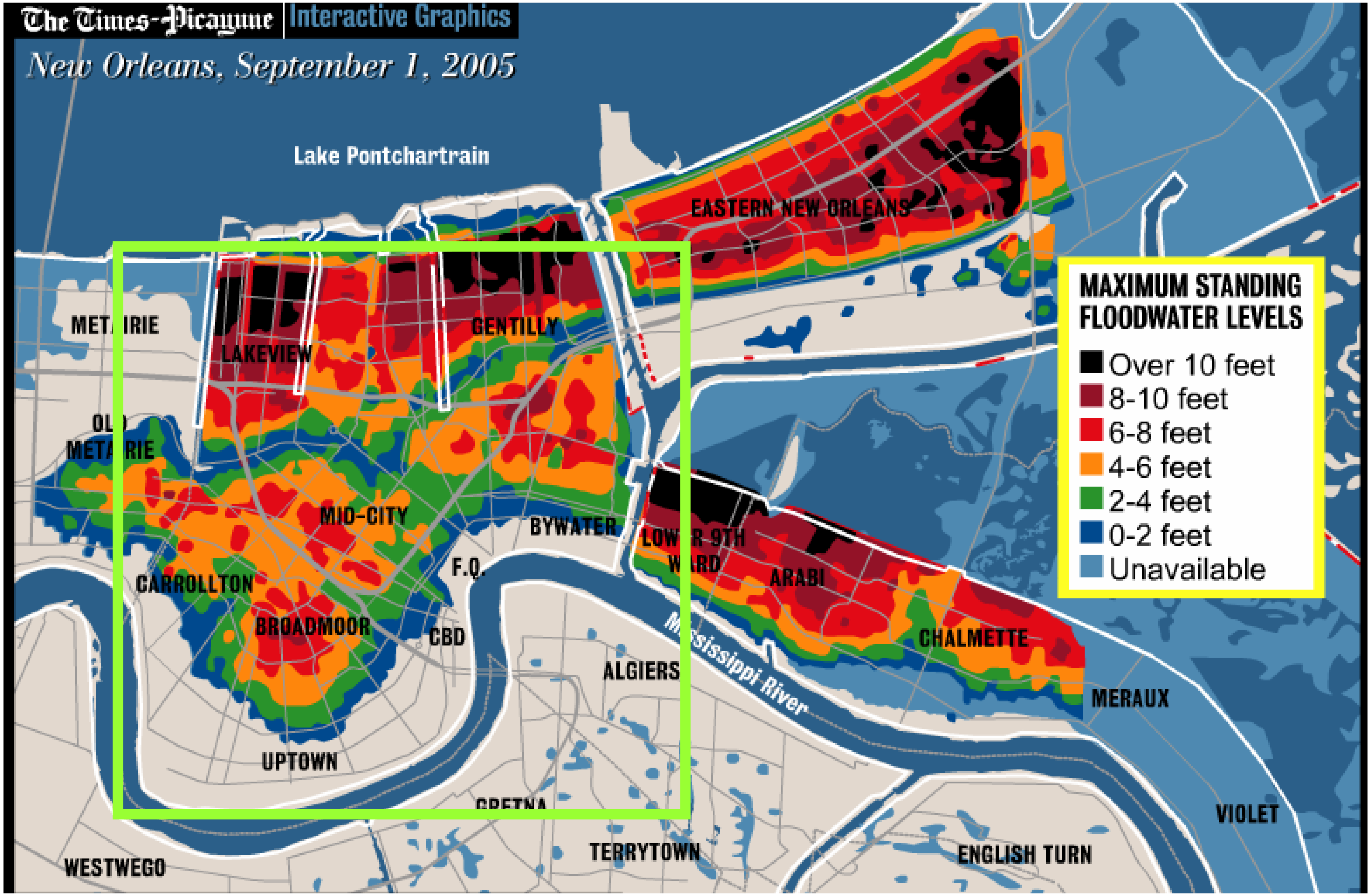
\# Districts = **5**

\# Passive district = **2**

	Land use	NSB
	Human	11
	Human	13
	Human	16
	Passive	9
	Passive	13

- Decisions consistent with modeling intuition
- Heuristic with weight (1, 1, 1, 1) solves less than a second.

Model Recommendations are consistent with Katrina Impact



Resource: http://www.nola.com/katrina/graphics/continuous.swf

Test Result Summary

- Unity weight test (all weights are equal)
  - All parcels in the same district are assigned to the same land use.
  - Parcels nearby water source are assigned to the Passive land use when their elevation is low.
  - Parcels in the same district are connected.
- Corner solutions
  - Non-zero weight on contiguity objective is necessary to prevent 'Checkerboard' pattern.
  - Spatial configurations consistent with objective formulations.

Conclusion & Contribution

- Reconstruction planning in post-disaster area needs to address spatial issues.
- Both exact optimization method and heuristic method are competitive.
- Neither exact optimization method nor heuristic algorithm has previously been used in post-disaster planning.
- Our mathematical planning model can provide a framework for the reconstruction strategies in post-disaster area.

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Future Research Direction

- Extend the problem to other areas where spatial configuration required.
- Investigate constraint reformulation that identifies facet-defining inequalities.
- Decompose to Two-phase districting problems.

Thank you!

Two-phase districting

- Phase I: Generating feasible districts
  - Use Tree search algorithm (Garfinkel and Nemhauser, 1970)
  - Use Column generation method (Mehrotra et al, 1998)
- Phase II: Optimization
  - Each neighborhood is included in one and only one district.
  - Total number of districts is pre-specified.

Reserve Design

- **Reserve design model (Williams *et al*, 2005)**

$$\text{Minimize} \quad w_1 \sum_{i\in I}\sum_{j>i} d_{ij}u_{ij} - w_2 \sum_{i\in I}\sum_{k>i} u_{ik}$$

$$+w_3\left(\sum_{i\in I} b_i x_i - 2\sum_{i\in I}\sum_{k>i} b_{ik}u_{ik}\right) + w_4 \sum_{i\in I} c_i x_i$$

$$\text{Subject to} \quad \sum_{i\in I} x_i = M$$

$$u_{ij} \geq x_i + x_j - 1$$

$$x_i, u_{ij} = binary$$

Number of parcels connected

Distance between selected parcels

Total boundary length

Total cost

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Land Acquisition

- **Land acquisition model (Wright *et al*, 1983)**

$$\text{Minimize} \quad \underbrace{w_b \sum_{i=1}^{n} c_i x_i}_{\text{Total cost}} + w_c \underbrace{\sum_{i=1}^{n} \sum_{j \in T_i} S_{ij}(P_{ij} + N_{ij})}_{\text{Total boundary length}}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} x_i = M$$

$$x_i - x_j - P_{ij} + N_{ij} = 0 \qquad \forall i, j \in T_i$$

$$x_i, P_{ij}, N_{ij} = binary \qquad \forall i, j \in T_i$$

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Political Districting

- **Political Districting model (Hess *et al*, 1965)**

$$\textit{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^{2} P_j x_{ij}$$

Minimize moment of inertia

$$\textit{Subject to} \quad \sum_{j=1}^{n} x_{ij} = 1 \qquad \forall i \in \{1,2,\ldots,n\}$$

$$\sum_{j=1}^{n} x_{jj} = M \qquad \forall j \in \{1,2,\ldots,n\}$$

$$\sum_{i=1}^{n} P_i x_{ij} \geq (1-\alpha)\left(\sum_{i=1}^{n} P_i \big/ M\right) x_{ij} \qquad \forall j \in \{1,2,\ldots,n\}$$

$$\sum_{i=1}^{n} P_i x_{ij} \leq (1+\alpha)\left(\sum_{i=1}^{n} P_i \big/ M\right) x_{ij} \qquad \forall j \in \{1,2,\ldots,n\}$$

$$x_{ij} \ \textit{binary} \qquad \forall i, j$$

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